Double Integrals

Just like we can find partial derivatives, we can find partial integrals. For example:

$$\int_1^2 xy^2 dx =$$

Notice that our result above is a function of ____.

What will the following partial integral be a function of? _____

$$\int_{-1}^1 xy^2 dy =$$

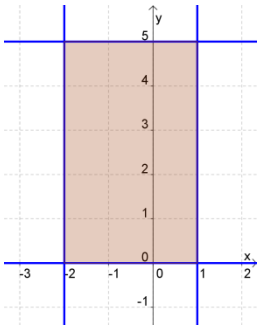
Now we can talk about the _____ $\iint_R f(x,y)dA$ over the rectangular region R, where $a \le x \le b$ and $c \le y \le d$.

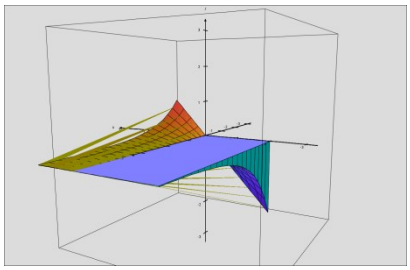
$$\iint_R f(x,y)dA = \int_a^b \left[\int_c^d f(x,y)dy\right]dx = \int_c^d \left[\int_a^b f(x,y)dx\right]dy$$

Ex 1.

Evaluate the following double integral:

$$\int_0^5 \int_{-2}^1 xe^{-y}\, dxdy$$





Ex 2.

Evaluate the following double integral:

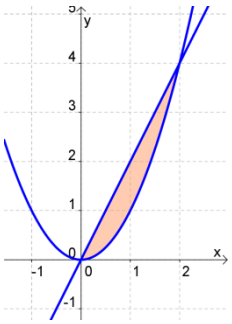
$$\int_{-2}^{1}\int_{0}^{5} xe^{-y}\,dydx$$

We can also define double integrals over some nonrectangular regions. Here's an example:

Ex 3.

Evaluate the following double integral:

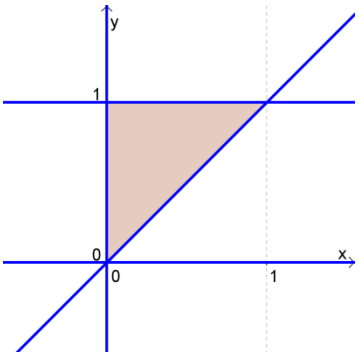
$$\int_{0}^{2}\int_{x^2}^{2x} xy\,dydx$$



Ex 4.

Evaluate the following double integral:

$$\int_0^1 \int_0^y y^2 e^{xy} dx dy$$



Practice

1. Evaluate the following double integral:

$$\int_0^1 \int_0^1 x^2 e^{xy}\, dydx$$

2. Evaluate the following double integral:

$$\int_{-3}^3 \int_0^{4x} (y - x)\, dydx$$

Q: April says May is a liar. May says June is a liar. June says April and May are both liars. If only one person is telling the truth, who is it?