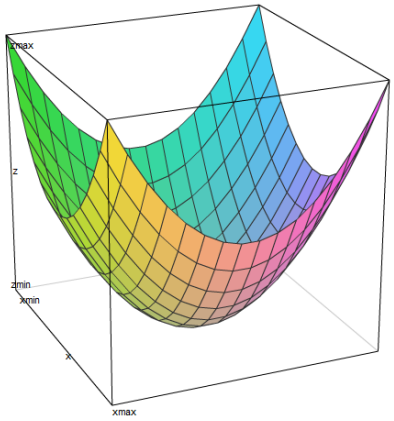
Optimizing Functions of Two Variables

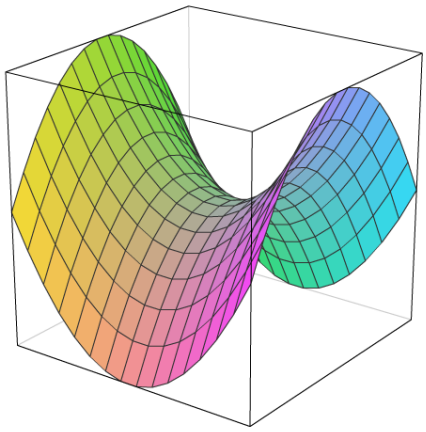
$f(x,y)$ has a _____ at (a,b) if $f(a,b) \geq f(x,y)$ for all (x,y) close to (a,b).

$f(x,y)$ has a _____ at (c,d) if $f(c,d) \leq f(x,y)$ for all (x,y) close to (c,d).

To help find relative extrema, we'll find _____, which are points where both $f_x = 0$ and $f_y = 0$.

Relative extrema *might* happen at critical points, but they might *not*. See examples below:





The Second Partials Test

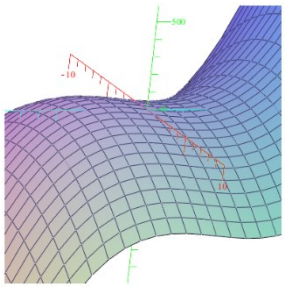
Let $\boldsymbol{D(x,y) = f_{xx}f_{yy} - (f_{xy})^2}$

1. Find $f_x, f_y, f_{xx}, f_{xy}, f_{yy}$, and $D(x,y)$.
2. Find all _____ of f.
3. For each critical point (a,b), evaluate _____.
4. If $D(a,b) < 0$, then there is a _____ at (a,b).
5. If $D(a,b) > 0$, then compute $f_{xx}(a,b)$.
   a. If $f_{xx}(a,b) > 0$, then there is a _____ at (a,b).
   b. If $f_{xx}(a,b) < 0$, then there is a _____ at (a,b).

(Note: If $D(a,b) = 0$, then test doesn't tell you anything.)

Ex 1.

Find all critical points for the function $f(x, y) = 12x - x^3 - 4y^2$, and classify each as a relative maximum, a relative minimum, or a saddle point.



Ex 2.

Find all critical points for the function $f(x, y) = x^3 - y^3 + 6xy$, and classify each as a relative maximum, a relative minimum, or a saddle point.

Practice

1. Find all critical points for the function $f(x,y) = xy + \frac{8}{x} + \frac{8}{y}$ and classify each as a relative maximum, a relative minimum, or a saddle point.

Q: A sheik announced that a race would decide which of his two sons would inherit all his wealth. The sons were to ride their camels to a certain distant city. The son whose camel reached the city last would be given all of the sheik's wealth. The two sons set out on the journey. After several days of aimless wandering, they met and agreed to seek the advice of a wiseman. After listening to the wiseman's advice, the two sons rode the camels as quickly as possible to the designated city. They did not agree to split the wealth, and their father's decree was to be followed. What was it the wiseman told the two sons?