

1. Find:

$$\int x e^{-3x} dx$$

$$= \underbrace{(x)}_u \underbrace{\left(-\frac{1}{3} e^{-3x}\right)}_v - \int \underbrace{\left(-\frac{1}{3} e^{-3x}\right)}_v \underbrace{dx}_{du}$$

$$u = x$$

$$du = dx$$

$$dv = e^{-3x} dx$$

$$v = -\frac{1}{3} e^{-3x}$$

$$= -\frac{1}{3} x e^{-3x} + \int \frac{1}{3} e^{-3x} dx$$

$$= \boxed{-\frac{1}{3} x e^{-3x} - \frac{1}{9} e^{-3x} + C}$$

2. Find:

$$\int_1^2 x \ln x^2 dx$$

$$= \int_1^2 2x \ln x dx$$

$$= \int_1^2 \underbrace{(\ln x)}_u \underbrace{(2x dx)}_{dv}$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$dv = 2x dx$$

$$v = x^2$$

$$= \underbrace{(\ln x)}_u \underbrace{(x^2)}_v \Big|_1^2 - \int_1^2 \underbrace{(x^2)}_v \underbrace{\left(\frac{1}{x} dx\right)}_{du}$$

$$= x^2 \ln x \Big|_1^2 - \int_1^2 x dx$$

$$= (2)^2 \ln(2) - (1)^2 \ln(1) - \left(\frac{x^2}{2}\right) \Big|_1^2$$

$$= 4 \ln 2 - \left(\frac{(2)^2}{2} - \frac{(1)^2}{2}\right)$$

$$= \boxed{4 \ln 2 - \frac{3}{2}}$$

Q: There is a word in the English language in which the first two letters signify a male, the first three letters signify a female, the first four signify a great man, and the whole word, a great woman. What is the word?