

Integration by Parts; Integral Tables

Just like substitution, **integration by parts** is a technique to turn a difficult integral into a simpler one. It is derived from the product rule for derivatives.

$$\frac{d}{dx} [u(x)v(x)] = u(x) \frac{dv}{dx} + v(x) \frac{du}{dx}$$

$$u(x) \frac{dv}{dx} = \frac{d}{dx} [u(x)v(x)] - v(x) \frac{du}{dx}$$

Integrate both sides to get:

$$\int u(x) \frac{dv}{dx} dx = u(x)v(x) - \int v(x) \frac{du}{dx} dx$$

We can clean it up to get the **integration by parts formula**:

$$\boxed{\int u dv = uv - \int v du}$$

Ex 1.

Find:

$$\int x^2 \ln x dx$$

Generally, pick u so du is simpler. And pick a dv that is easy to integrate.

Ex 2.

Find:

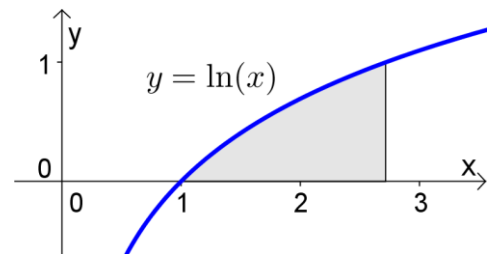
$$\int x\sqrt{x+5} dx$$

For definite integrals (ones that have limits of integration), use:

$$\int_a^b u \, dv = uv \Big|_a^b - \int_a^b v \, du$$

Ex 3.

Find the area under $y = \ln x$ between $x = 1$ to $x = e$.

**Ex 4.**

Find:

$$\int x^2 e^{2x} \, dx$$

Practice

1. Find: $\int x e^{-3x} dx$

2. Find: $\int_1^2 x \ln x^2 dx$

Q: There is a word in the English language in which the first two letters signify a male, the first three letters signify a female, the first four signify a great man, and the whole word, a great woman. What is the word?