

1. Differentiate:

a) $f(x) = \frac{e^{5x}}{x+1}$

$$\begin{aligned}
 f'(x) &= \frac{(x+1)(e^{5x} \cdot 5) - (e^{5x})(1)}{(x+1)^2} \\
 &= \frac{e^{5x}((x+1) \cdot 5 - 1)}{(x+1)^2} \\
 &= \boxed{\frac{e^{5x}(5x+4)}{(x+1)^2}}
 \end{aligned}$$

Note: Could also use
logarithmic differentiation.

$$\begin{aligned}
 \ln f(x) &= \ln \frac{e^{5x}}{x+1} \\
 &= \ln e^{5x} - \ln(x+1) \\
 &= 5x - \ln(x+1)
 \end{aligned}$$

$$\begin{aligned}
 \frac{1}{f(x)} \cdot f'(x) &= 5 - \frac{1}{x+1} \\
 f'(x) &= \boxed{\frac{e^{5x}}{x+1} \left(5 - \frac{1}{x+1}\right)}
 \end{aligned}$$

b) $f(x) = x \ln \sqrt{x^2-3} = x \ln(x^2-3)^{1/2} = \frac{1}{2}x \cdot \ln(x^2-3)$

$$f'(x) = \left(\frac{1}{2}x\right) \left(\frac{1}{x^2-3} \cdot 2x\right) + \left[\ln(x^2-3)\right] \left(\frac{1}{2}\right)$$

$$= \boxed{\frac{x^2}{x^2-3} + \frac{1}{2} \ln(x^2-3)}$$

c) $f(x) = \log_3(5x)$

$$f'(x) = \frac{1}{5x \ln 3} \cdot 5$$

$$= \boxed{\frac{1}{x \ln 3}}$$

2. Use logarithmic differentiation to find the derivative of the following function.

$$f(x) = x^2 e^{-3x} (2x + 5)^5$$

$$\begin{aligned} \ln f(x) &= \ln x^2 e^{-3x} (2x+5)^5 \\ &= \ln x^2 + \ln e^{-3x} + \ln (2x+5)^5 \\ &= \underline{2 \ln x - 3x + 5 \ln (2x+5)} \end{aligned}$$

Differentiate both sides:

$$\frac{1}{f(x)} \cdot f'(x) = 2 \cdot \frac{1}{x} - 3 + 5 \cdot \frac{1}{2x+5} \cdot 2$$

$$\frac{f'(x)}{f(x)} = \frac{2}{x} - 3 + \frac{10}{2x+5}$$

$$f'(x) = \boxed{f(x) \left[\frac{2}{x} - 3 + \frac{10}{2x+5} \right]} \quad \left(= x^2 e^{-3x} (2x+5)^5 \left[\frac{2}{x} - 3 + \frac{10}{2x+5} \right] \right)$$

Q: What has four wheels and flies?