1. Find the domain, intercepts, asymptotes, relative extrema, and inflection points, and sketch the graph of $f(x) = \frac{2x^2}{x^2-1}$

① Domain: all #'s except 1 and −1

② x-int (set $y=0$): $2x^2=0 \to x=0$ $(0,0)$

y-int (set $x=0$): $f(0)=0$ $(0,0)$

③ V.A: $x=-1$ (↑ top $\neq 0$) $x=1$ (↑ top $\neq 0$)

H.A: $\lim_{x\to\pm\infty} \frac{2x^2}{x^2-1} = \lim_{x\to\pm\infty} \frac{2}{1-\frac{1}{x^2}} = 2$

So, H.A. at $y=2$

(same for $x \to -\infty$)

④ $f'(x) = \frac{(x^2-1)(4x)-(2x^2)(2x)}{(x^2-1)^2} = \frac{4x^3-4x-4x^3}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2}$

$f'(x)=0$: $-4x=0 \to x=0$

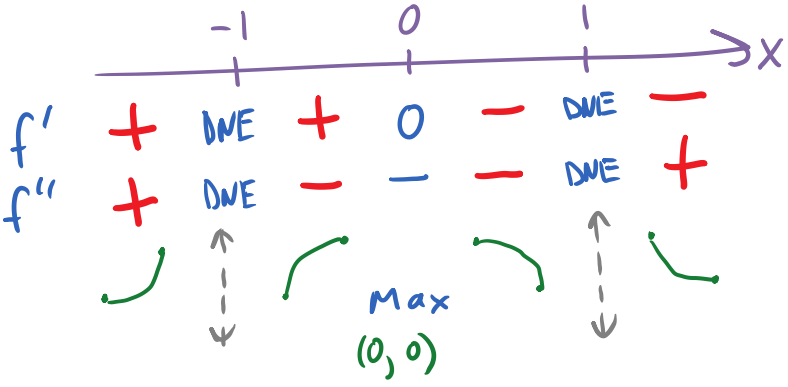
$f'(x)$ DNE: $(x^2-1)^2=0 \to x=-1, x=1$

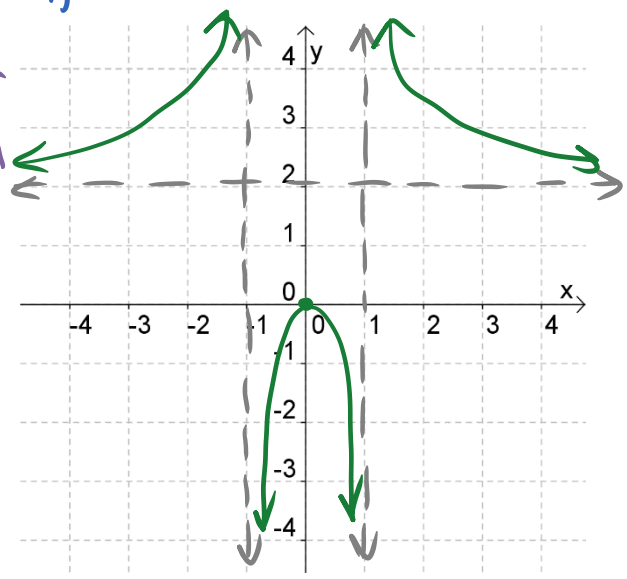
$f''(x) = \frac{(x^2-1)^2(-4)-(-4x)(2(x^2-1)(2x))}{(x^2-1)^4} = \frac{-4(x^2-1)\left[(x^2-1)-x\cdot 2\cdot 2x\right]}{(x^2-1)^{4\,3}}$

$= \frac{-4(x^2-1-4x^2)}{(x^2-1)^3} = \frac{4(3x^2+1)}{(x^2-1)^3}$

$f''(x)=0$: $4(3x^2+1)=0 \to$ never

$f''(x)$ DNE: $(x^2-1)^3=0 \to x=1, x=-1$





Q: What belongs to you but others use it more than you do?