

1. Determine the intervals of concavity for $f(x) = x^4 + 6x^3 - 24x^2 + 2$, and find all inflection points.

$$f'(x) = 4x^3 + 18x^2 - 48x$$

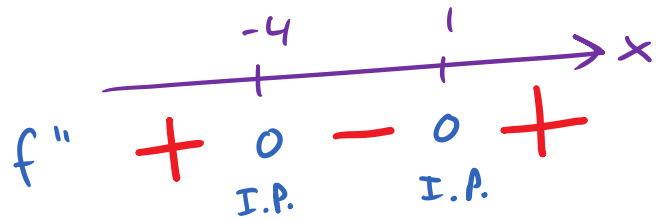
$$\begin{aligned} f''(x) &= 12x^2 + 36x - 48 \\ &= 12(x^2 + 3x - 4) \\ &= 12(x+4)(x-1) \end{aligned}$$

$$\underline{f''(x) = 0:}$$

$$\begin{array}{ccc} 12(x+4)(x-1) = 0 & & \\ \downarrow & & \downarrow \\ x = -4 & & x = 1 \end{array}$$

$$\underline{f''(x) \text{ DNE:}}$$

Nowhere



Concave up: $x < -4, x > 1$

Concave down: $-4 < x < 1$

Inflection points:

$(-4, -510), (1, -15)$

2. Use the 2nd Derivative Test to find the relative maxima and minima of $f(x) = 2x + 1 + \frac{2}{x}$.

$$f'(x) = 2 - \frac{2}{x^2}$$

$$f''(x) = \frac{4}{x^3}$$

$$\underline{f'(x) = 0:}$$

$$2 - \frac{2}{x^2} = 0$$

$$2 = \frac{2}{x^2}$$

$$2x^2 = 2$$

$$x^2 = 1$$

$$x = \pm 1$$

$$(-1, -3) \quad (1, 5)$$

$$\uparrow$$

$$f(-1)$$

$$\uparrow$$

$$f(1)$$

2nd Derivative Test:

$$f''(-1) = \frac{4}{(-1)^3} = -4 < 0$$

So, $(-1, -3)$ is a relative max

$$f''(1) = \frac{4}{(1)^3} = 4 > 0$$

So, $(1, 5)$ is a relative min

Q: What holds water yet is full of holes?