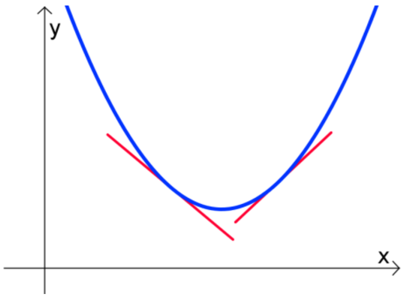
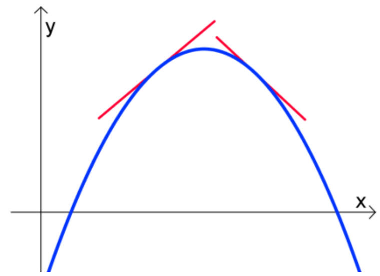
Concavity and Points of Inflection

The 2nd derivative tells us how the 1st derivative is changing.

If f'' is positive, then f' is ____________________, and the graph of f is ______________________.

If f'' is negative, then f' is ____________________, and the graph of f is ______________________.



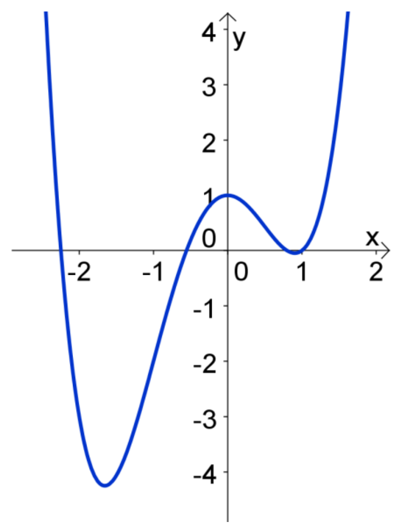


Where might $f(x)$ change from concave up to concave down, or concave down to concave up?

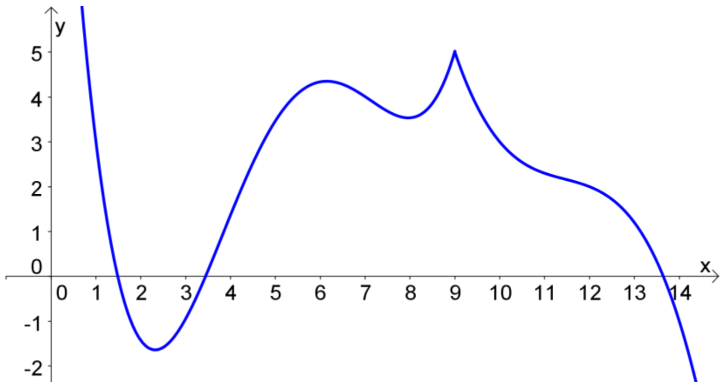
1. When $f''(x) = 0$
2. When $f''(x)$ does not exist (DNE)

Ex 1.

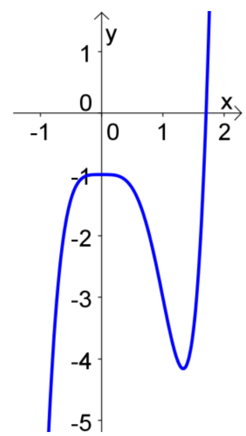
Determine the intervals of concavity for $f(x) = x^4 + x^3 - 3x^2 + 1$.



A point $(c, f(c))$ where the concavity changes is called an ________________________.
(Again, this might happen if either $f''(c) = 0$ or $f''(c)$ does not exist.)

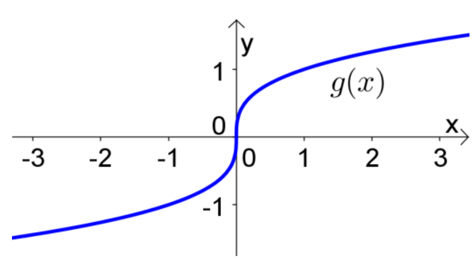


Ex 2.
Find all inflection points of $f(x) = 3x^5 - 5x^4 - 1$.



Ex 3.

Find all inflection points of $g(x) = x^{\frac{1}{3}}$.



2^{nd} Derivative Test

Suppose $f'(c) = 0$.

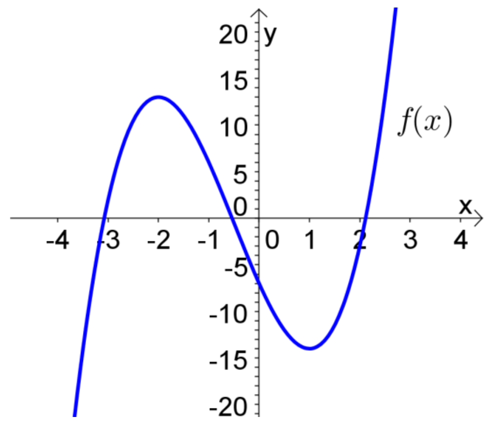
If $f''(c) > 0$, then there is a relative ________________at $x = c$.

If $f''(c) < 0$, then there is a relative ________________at $x = c$.

If $f''(c) = 0$ or $f''(c)$ does not exist, then test doesn't say anything (maybe try 1^{st} Derivative Test).

Ex 4.

Use the 2^{nd} Derivative Test to find the relative maxima and minima of $f(x) = 2x^3 + 3x^2 - 12x - 7$.



Note:

$f(x) = x^4$

$f'(x) = 4x^3$

$f''(x) = 12x^2$

Note that here, $f'(x) = 0$ when $x = 0$, but $f''(0) = 0$, so the 2^{nd} Derivative Test is inconclusive.

Summary:

1^{st} Derivative Test – Uses sign of $\boldsymbol{f'}$ **across** a critical number to find relative max/min.

2^{nd} Derivative Test – Uses sign of $\boldsymbol{f''}$ **at** a critical number to find relative max/min.

Practice

1. Determine the intervals of concavity for $f(x) = x^4 + 6x^3 - 24x^2 + 2$, and find all inflection points.

2. Use the 2nd Derivative Test to find the relative maxima and minima of $f(x) = 2x + 1 + \frac{2}{x}$.

Q: What holds water yet is full of holes?