The Chain Rule

ex: If a car uses 0.2 gallons per mile when travelling 60 miles per hour, how many gallons per hour are being used?

ex: The cost to make a product (C) depends on the number of units made (q).
The number of units made (q) depends on time (t).

$\frac{dC}{dq}$ represents the rate of change of **cost** with respect to **output** (ex: $200 per unit)

$\frac{dq}{dt}$ represents the rate of change of **output** with respect to **time** (ex: 10 units per hour)

$\frac{dC}{dt}$ represents the rate of change of **cost** with respect to **time** (ex: ____________________)

$\frac{dC}{dt} =$

Ex 1.
Differentiate $y = (2x - 5)^2$.

The Chain Rule
If $y = f(u)$ is a differentiable function of u,
and $u = g(x)$ is a differentiable function of x,
then $y = f\big(g(x)\big)$ is a differentiable function of x, and

$$\boldsymbol{\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}}$$

Also written $\frac{dy}{dx} = f'\big(g(x)\big)g'(x)$

Ex 2.

Given $y = \frac{u}{u+1}$ and $u = 3x^2 - 1$, find $\frac{dy}{dx}$ at $x = 1$.

Ex 3.

The cost of producing x units is $C(x) = \frac{1}{3}x^2 + 4x + 53$ dollars, and production level t hours into a particular production run is $x(t) = 0.2t^2 + 0.03t$ units. At what rate is cost changing with respect to time after 4 hours?

Inner/Outer Functions

Recall this version of the Chain Rule: If $y = f\left(g(x)\right)$, then $\frac{dy}{dx} = f'\left(g(x)\right)g'(x)$.

Ex 4.

Differentiate $y = \sqrt{x^2 - 3x + 5}$

Ex 5.

Differentiate $h(x) = (2x^3 - 3x)^4$

Ex 6.

Differentiate $f(x) = \frac{1}{(2x+3)^5}$

Ex 7.

Differentiate $f(x) = (3x + 1)^4(2x - 1)^5$ and simplify your answer by factoring.

Practice

1. Differentiate $y = -2(3x^2 + 2)^5$

2. Suppose $y = \sqrt{u}$ and $u = x^2 + 3x - 1$.

a) Find $\frac{dy}{dx}$

b) Find $\frac{dy}{dx}$ at $x = 2$.

3. Differentiate $f(x) = \frac{1}{(2-5x)^6}$

4. Differentiate $f(x) = (2x - 1)^3(5x + 3)^4$ and simplify your answer by factoring.

Q: What five-letter word becomes shorter when you add two letters to it?