Product and Quotient Rules; Higher-Order Derivatives

Here are two more shortcuts to computing the derivative.

Product Rule: $\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$

In other words, $(fg)' = fg' + gf'$

Ex 1.
Differentiate $f(x) = (x-3)(2x^2+4x-1)$.

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$

In other words, $\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$

Ex 2.
Differentiate $f(x) = \frac{x^2-3x+5}{2x-1}$.

Notation: A shorter way to write $\frac{dy}{dx}$ is y'.

The Second Derivative

$f''(x) = \frac{d}{dx}[f'(x)]$ also written $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$ or $y'' = (y')'$

Ex 3.

Find the second derivative of $y = 2x^4 - 5x^2 + 23x - 10$

The nth Derivative

To get the third derivative, fourth derivative, fifth derivative, etc, just keep differentiating. For example, here are the derivatives of $f(x) = 4x^3 - 2x^2 + 5x - 1$:

First derivative: $f'(x) = 12x^2 - 4x + 5$
Second derivative: $f''(x) = 24x - 4$
Third derivative: $f'''(x) = 24$
Fourth derivative: $f^{(4)}(x) = 0$
Fifth derivative: $f^{(5)}(x) = 0$

Practice

1. Differentiate $f(x) = (x+3)(2x-5)$

2. Differentiate $y = \frac{x^2+1}{1-x^2}$

3. Find an equation for the tangent line to $y = \frac{x}{2x+1}$ at the point where $x = -2$. (Hint: Find y', then find $y'(-2)$ to find the slope of the tangent line at $x = -2$, then use point-slope form.)

4. Find all points on the graph of $f(x) = (x-1)(x^2 - 3x + 2)$ where the tangent line is horizontal. (Hint: Find $f'(x)$, then find all x where $f'(x) = 0$.)

5. Find the second derivative of $y = x^2(3x + 2)$.

6. Find the fifth derivative of $y = \frac{1}{x}$ (Hint: Rewrite $\frac{1}{x}$ as x^{-1}, then use Power Rule to take derivatives.)

Q: I can run but not walk. Wherever I go, thought follows close behind. What am I?