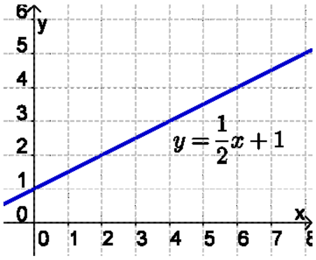
The Derivative

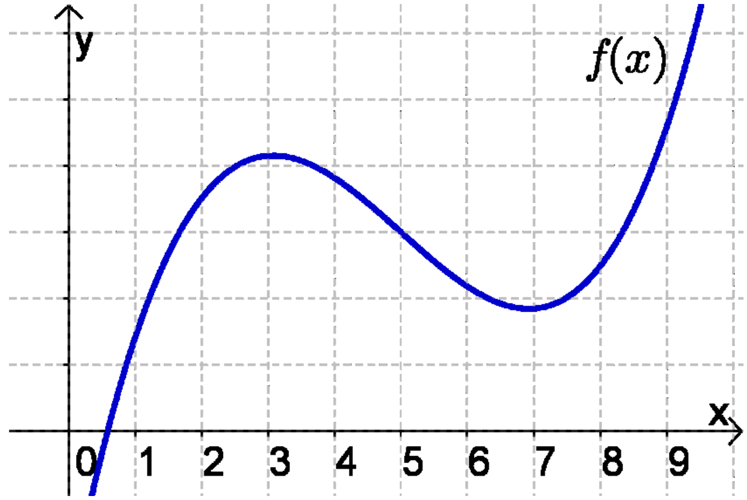


The line to the left has a constant rate of change, measured by the slope, $\frac{1}{2}$.

For nonlinear functions (like the graph to the right), we use the

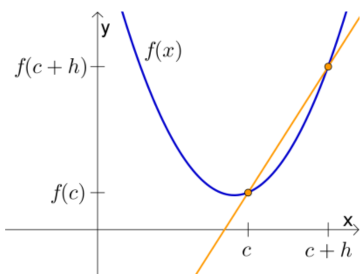
____

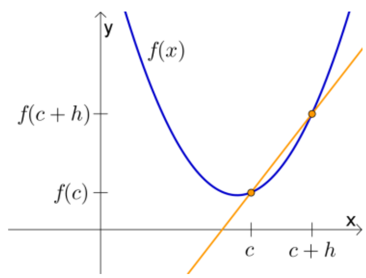
to measure "instantaneous" rate of change.

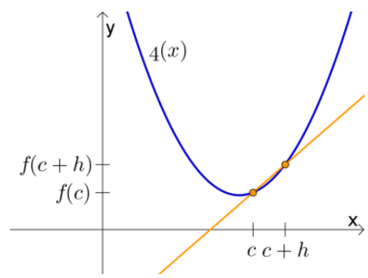


Q: Given $f(x)$, how do we find the slope of the tangent line at any point $x = c$?

A: We look at the slopes of the ______________________ that approach the tangent line.







$$\text{slope of secant line} = \frac{\Delta y}{\Delta x} =$$

$$\text{slope of the tangent line} =$$

In general, the slope of the tangent line of $f(x)$ at any point x is called the ______________________, and is defined like this:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Notes:

$f'(x)$ is read "f prime of x"

$\frac{f(x+h)-f(x)}{h}$ is called the ___________________________.

_______________________________ is the process of computing the derivative.

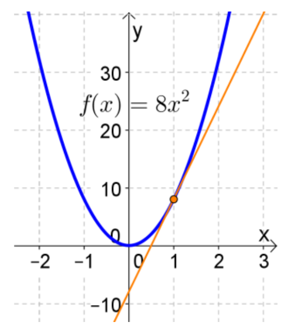
f is ______________________________________ if $f'(c)$ exists.

Ex 1.

Using the definition, find the derivative of $f(x) = 8x^2$.

Ex 2.

Find the slope and equation of the tangent line to $f(x) = 8x^2$ at $x = 1$.



Notes:

If $f'(c) > 0$, then f is ________________________ at $x = c$.

If $f'(c) < 0$, then f is ________________________ at $x = c$.

Another way to write the derivative is: $\frac{dy}{dx}$

ex: Suppose $y = 8x^2$. Then $\frac{dy}{dx} = 16x$.

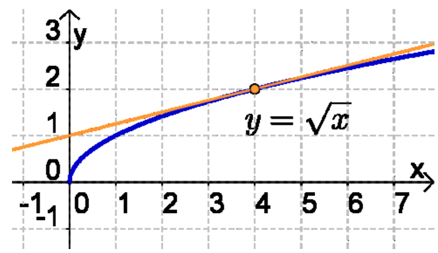
Also, $\left.\frac{dy}{dx}\right|_{x=-3}$ means "$\frac{dy}{dx}$ evaluated at $x = -3$". So,

$$\left.\frac{dy}{dx}\right|_{x=-3} =$$

Ex 3.

Using the definition, find the derivative of $y = \sqrt{x}$.

Now find the slope-intercept equation of the tangent line to the curve $y = \sqrt{x}$ at $x = 4$.



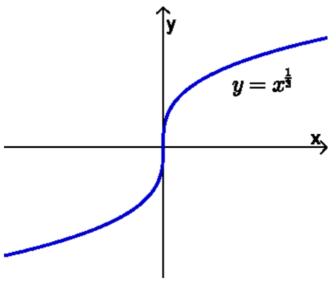
Now find the rate of change of $y = \sqrt{x}$ where $x = 1$.

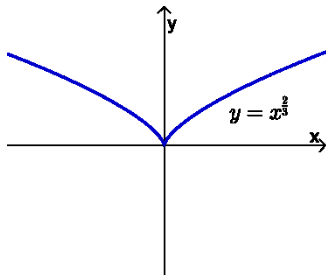
Based on the previous example, if $f(x) = \sqrt{x}$, then the derivative is $f'(x) = \frac{1}{2\sqrt{x}}$

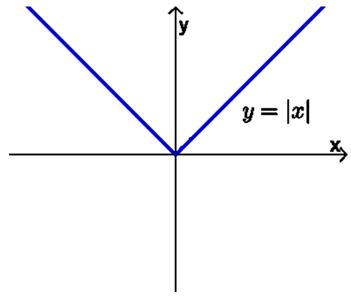
Note that $f(0)$ is defined (it's 0), but $f'(0)$ is undefined (so f is not differentiable at $x = 0$).

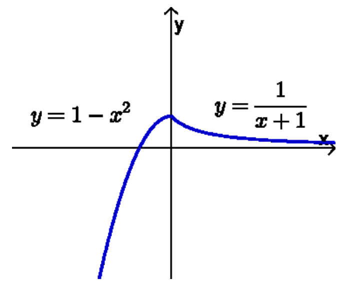
So, the domain of the derivative might be different than the original function.

Here are some continuous functions that are not differentiable at $x = 0$:









Practice

1. a) Using the definition, find the derivative of $f(x) = x^2 + x$.

   b) Now find the slope and equation of the line that is tangent to $f(x) = x^2 + x$ at $x = 1$.

   c) Is $f(x) = x^2 + x$ increasing or decreasing at $x = 1$?

2. a) Using the definition, find the derivative of $y = \frac{1}{x}$ (Hint: when you have fractions of fractions, remember you can multiply top and bottom by the LCD)

b) Now find the rate of change of $y = \frac{1}{x}$ where $x = 2$.

Q: What has a head and a tail, but no body?