Test #1 Name: Solutions

Math 130, Section 21, David Beydler Wednesday, September 28, 2011

Directions: Show all work to get full credit. No calculators, books, notes. Please box your answers. Good luck!
(60 points total)

1. (3 points) You want to strengthen a mixture that is 10% alcohol to one that is 40% alcohol. How much pure alcohol should be added to 20 L of the 10% mixture?

Let x = liters of pure alcohol.

$\boxed{10 \text{ L of pure alcohol}}$

$$\underbrace{(0.1)(20)}_{\text{alcohol in 10\% mix.}} + \underbrace{x}_{\text{pure alcohol}} = \underbrace{0.4(20+x)}_{\text{alcohol in 40\% mix}}$$

$$2 + x = 8 + 0.4x$$

$$0.6x = 6$$

$$x = \frac{6}{0.6} = \frac{60}{6} = 10$$

2. (3 points) You deposit some money at 10% interest, and deposit one third as much at 24%. Find the amount deposited at each rate if the total annual interest income is $54.

Let x = \$ deposited at 10% interest

$$\underbrace{0.1x}_{\text{interest income at 10\%}} + \underbrace{0.24\left(\tfrac{1}{3}x\right)}_{\text{interest income at 24\%}} = \underbrace{54}_{\text{total interest income}}$$

$$0.1x + 0.08x = 54$$

$$0.18x = 54$$

$$x = \frac{54}{0.18} = \frac{5400}{18} = 300$$

$\boxed{\begin{array}{l}\$300 \text{ at } 10\% \\ \$100 \text{ at } 24\%\end{array}}$

$\left(\frac{1}{3}\cdot 300\right)$

3. (2 points) Solve the equation $x(x+6) = -10$. Simplify any radicals.

$$x^2 + 6x = -10$$

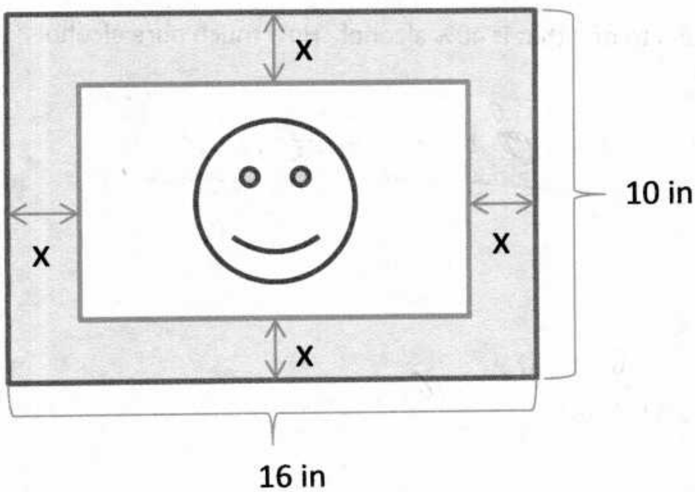
$$x^2 + 6x + 10 = 0$$

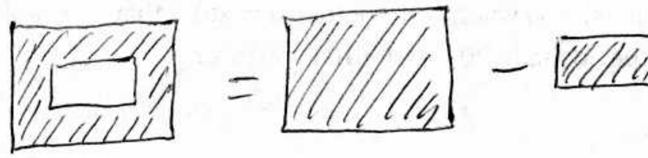
$$x = \frac{-6 \pm \sqrt{36-40}}{2}$$

$$= \frac{-6 \pm \sqrt{-4}}{2} = \frac{-6 \pm 2i}{2} = -3 \pm i$$

$\boxed{\{-3 \pm i\}}$

4. (3 points) You have a painting in a frame that is 16 inches long and 10 inches high. Suppose the area of the frame (shaded in the picture below) is 88 [redacted] square inches, and the frame has a uniform width, call it x. What is the width, x?





$$\boxed{\text{Width is } 2\text{ in}}$$

$88 [redacted] = (16)(10) - (16-2x)(10-2x)$

$88 [redacted] = 160 - (160 - 52x + 4x^2)$

$88 [redacted] = 52x - 4x^2$

$22 [redacted] = 13x - x^2$

$x^2 - 13x + 22 = 0$

$(x-11)(x-2) = 0$

$\cancel{x=11}$ or $x = 2$

Doesn't make sense in problem

5. (3 points) Solve $\sqrt{x+3} - \sqrt{3x+10} = 1$.

$\sqrt{x+3} = 1 + \sqrt{3x+10}$

$x+3 = 1 + 2\sqrt{3x+10} + 3x+10$

$-2x-8 = 2\sqrt{3x+10}$

$-x-4 = \sqrt{3x+10}$

$x^2+8x+16 = 3x+10$

$x^2+5x+6=0$

$(x+2)(x+3)=0$

$x=-2 \quad x=-3$

Solution set: $\boxed{\emptyset}$

Check:

$x=-2$:

$\sqrt{-2+3} - \sqrt{3(-2)+10} \stackrel{?}{=} 1$

$1 - 2 \stackrel{?}{=} 1$

$-1 = 1 \leftarrow$ False, so $x=-2$ not a solution.

$x=-3$:

$\sqrt{-3+3} - \sqrt{3(-3)+10} \stackrel{?}{=} 1$

$0 - 1 \stackrel{?}{=} 1$

$-1 = 1 \leftarrow$ False, so $x=-3$ not a solution.

6. (3 points) Solve $\frac{3}{x} - \frac{x}{x+1} = -\frac{1}{x^2+x}$.

Factor

$$\frac{3}{x} - \frac{x}{x+1} = \frac{-1}{x(x+1)}$$

Multiply by $x(x+1)$

$$3(x+1) - x^2 = -1$$
$$3x + 3 - x^2 = -1$$
$$-x^2 + 3x + 4 = 0$$
$$x^2 - 3x - 4 = 0$$
$$(x-4)(x+1) = 0$$
$$x = 4 \qquad x = -1$$

Solution set: $\boxed{\{4\}}$

Check:

$x = 4$:

$$\frac{3}{4} - \frac{4}{4+1} \stackrel{?}{=} -\frac{1}{4^2+4}$$
$$\frac{3}{4} - \frac{4}{5} \stackrel{?}{=} \frac{-1}{20}$$
$$\frac{-1}{20} = \frac{-1}{20} \checkmark$$

$x = -1$:

$$\frac{3}{-1} - \frac{(-1)}{(-1)+1} = \frac{-1}{(-1)^2+(-1)}$$

Division by 0 undefined, so $x = -1$ not a solution

7. (3 points) Solve $\sqrt[3]{5x^2 - 6x + 2} - \sqrt[3]{x} = 0$

$$\sqrt[3]{5x^2 - 6x + 2} = \sqrt[3]{x}$$
$$5x^2 - 6x + 2 = x$$
$$5x^2 - 7x + 2 = 0$$
$$(5x-2)(x-1) = 0$$
$$x = \frac{2}{5} \qquad x = 1$$

Solution set: $\boxed{\left\{\frac{2}{5}, 1\right\}}$

8. (3 points) Solve $2x^{-4} - x^{-2} - 6 = 0$.

$u = x^{-2}$

$$2u^2 - u - 6 = 0$$
$$(2u+3)(u-2) = 0$$
$$u = \frac{-3}{2} \qquad u = 2$$
$$x^{-2} = \frac{-3}{2} \qquad x^{-2} = 2$$

$$x^2 = \frac{-2}{3} \qquad x^2 = \frac{1}{2}$$
$$x = \pm\sqrt{\frac{-2}{3}} \qquad x = \pm\sqrt{\frac{1}{2}}$$
$$x = \pm i\frac{\sqrt{2}}{\sqrt{3}} \qquad x = \pm\frac{1}{\sqrt{2}}$$
$$x = \pm i\frac{\sqrt{6}}{3} \qquad x = \pm\frac{\sqrt{2}}{2}$$

Solution set: $\boxed{\left\{\pm i\frac{\sqrt{6}}{3}, \pm\frac{\sqrt{2}}{2}\right\}}$

9. (3 points) Solve the rational inequality and write the solution set in interval notation.

$\frac{1}{3x-2} < 1$

$\frac{1}{3x-2} - 1 < 0$

$\frac{1}{3x-2} - \frac{3x-2}{3x-2} < 0$

$\frac{1-(3x-2)}{3x-2} < 0$

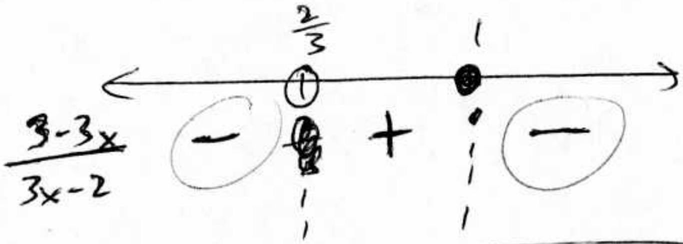
$\frac{1-3x+2}{3x-2} < 0$

$\frac{3-3x}{3x-2} < 0$

$\frac{3-3x}{3x-2}$ changes sign when

$3-3x=0$ or $3x-2=0$

$x=1$ $\quad$ $x=\frac{2}{3}$



Solution set $\boxed{(-\infty, \frac{2}{3}) \cup [1, \infty)}$

10. (2 points) Find the distance from $P(2\sqrt{3},\ -\sqrt{2})$ to $Q(-\sqrt{3},\ 3\sqrt{2})$.

$$\text{Distance} = \sqrt{(2\sqrt{3}-(-\sqrt{3}))^2+(-\sqrt{2}-3\sqrt{2})^2}$$
$$= \sqrt{(3\sqrt{3})^2+(-4\sqrt{2})^2}$$
$$= \sqrt{27+32} = \boxed{\sqrt{51}}$$

11. (3 points) Given that the midpoint of a segment is $(-2,3)$, and one endpoint is $(6,-2)$, find the coordinates for the other endpoint.

$\frac{x+6}{2} = -2$

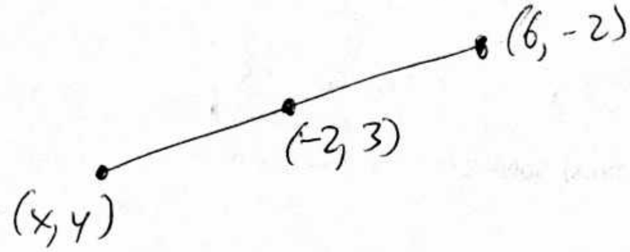
$x+6=-4$

$x=-10$

$\frac{y+(-2)}{2} = 3$

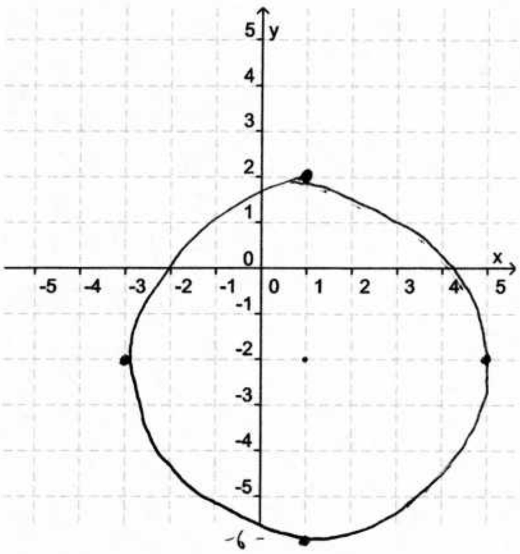
$y-2=6$

$y=8$



$\boxed{(-10, 8)}$

12. (3 points) Find the center-radius form of the equation of a circle with center $(1, -2)$ and radius 4. Then graph it below.



(4^2)

$$\boxed{(x-1)^2+(y+2)^2=16}$$

13. (3 points) Decide whether or not the following equation has a circle as its graph. If it does, give the center and the radius. If it does not, describe the graph.

$x^2 + y^2 - 6x - 6y + 18 = 0$

$$(x^2-6x+9)+(y^2-6y+9)=-18+9+9$$

$\left(\frac{-6}{2}\right)^2$ $\left(\frac{-6}{2}\right)^2$

$$(x-3)^2+(y-3)^2=0$$

$\boxed{\text{Not a circle}}$ $\boxed{\text{Graph is a point at } (3,3)}$

14. Consider the relation given by the equation $xy = -1$.

a. (1 point) Does the above relation define y as a function of x?

$\boxed{\text{Yes}}$

b. (1 point) What is the domain of the relation?

$\boxed{(-\infty,0)\cup(0,\infty)}$

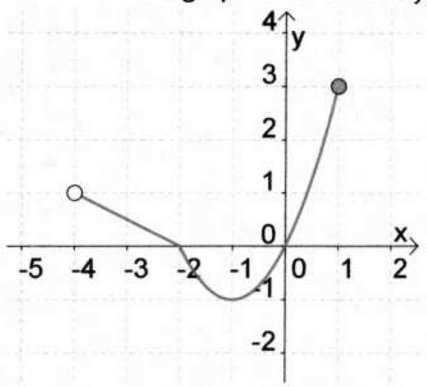
c. (1 point) What is the range of the relation?

$\boxed{(-\infty,0)\cup(0,\infty)}$

15. (2 points) Given that $f(x) = x^2 - 4x$, find $f(x+2)$.

$$f(x+2) = (x+2)^2 - 4(x+2)$$
$$= x^2 + 4x + 4 - 4x - 8$$
$$= \boxed{x^2 - 4}$$

16. Here's the graph of a function $f(x)$.



a. (1 point) What is the domain of the function? Write your answer using **interval notation**.

$\boxed{(-4, 1]}$

b. (1 point) What is the range of the function? Write your answer using **interval notation**.

$\boxed{[-1, 3]}$

c. (1 point) Find $f(-1)$.

$f(-1) = \boxed{-1}$

d. (1 point) For what value(s) of x is $f(x) = 3$?

$x = \boxed{1}$

e. (1 point) Find the interval(s) where $f(x)$ is increasing. Write your answer using **interval notation**.

$\boxed{[-1, 1]}$

f. (1 point) Find the interval(s) where $f(x)$ is decreasing. Write your answer using **interval notation**.

$\boxed{(-4, -1]}$

17. (3 points) Write the equation in slope-intercept form for the line that passes through $(2, -1)$ and is perpendicular to $3x - 2y = 4$.

$3x - 2y = 4$

$-2y = -3x + 4$

$y = \frac{3}{2}x - 2$

↑

slope: $\frac{3}{2}$

Perpendicular line

slope: $-\frac{2}{3}$

point: $(2, -1)$

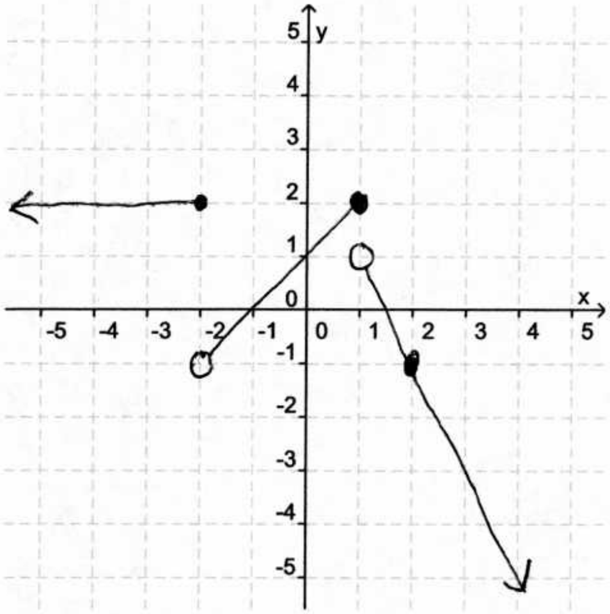
$y - (-1) = -\frac{2}{3}(x - 2)$

$y + 1 = -\frac{2}{3}x + \frac{4}{3}$

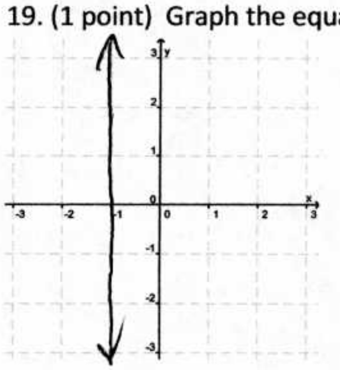
$\boxed{y = -\frac{2}{3}x + \frac{1}{3}}$

18. (3 points) Graph the following function:

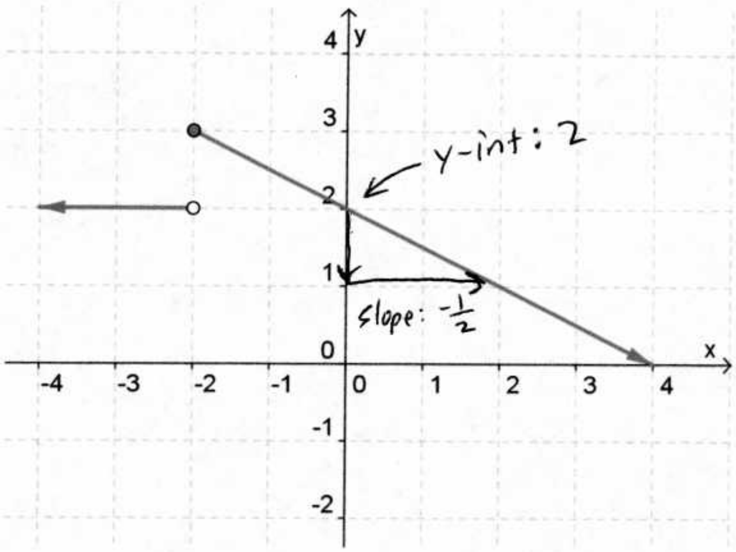
$$f(x) = \begin{cases} 2, & \text{if } x \le -2 \\ x+1, & \text{if } -2 < x \le 1 \\ -2x+3, & \text{if } x > 1 \end{cases}$$



19. (1 point) Graph the equation $x = -1$ on the rectangular coordinate system below.



20. Here's a piecewise-defined function.



a. (2 points) Give a rule for the function.

$$\boxed{f(x) = \begin{cases} 2, & \text{if } x < -2 \\ -\frac{1}{2}x + 2, & \text{if } x \geq -2 \end{cases}}$$

b. (2 points) Give the domain and range of the function.

Domain: $(-\infty, \infty)$

Range: $(-\infty, 3]$

c. (1 point) At what x-value(s) is the function discontinuous?

$x = \boxed{-2}$