

Quiz #5

Math 130, Section 21, David Beydler

Name: Solutions
 Wednesday, October 19, 2011

Directions: Show all work to get full credit. No calculators, books, notes. Please box your answers. Good luck!
 (15 points total)

1. (2 points) Use synthetic division to perform the following division:

$$\frac{-9x^3 + 8x^2 - 7x + 2}{x-2} = \boxed{-9x^2 - 10x - 27 - \frac{52}{x-2}}$$

$$\begin{array}{r|rrrr} 2 & -9 & 8 & -7 & 2 \\ & & -18 & -20 & -54 \\ \hline & -9 & -10 & -27 & -52 \end{array}$$

2. (2 points) Use synthetic division to decide whether $k = 1 - i$ is a zero of the function $f(x) = x^2 - 2x + 2$.

$$\begin{array}{r|rrr} 1+i & 1 & -2 & 2 \\ & & 1+i & -2 \\ \hline & 1 & -1+i & 0 \end{array} \leftarrow \text{Remainder is } 0,$$

so yes ($1-i$ is a zero of $f(x)$)

3. (2 points) Given that $k = -3$ is a zero of $f(x) = 6x^3 + 13x^2 - 14x + 3$, factor $f(x)$ into linear factors.

$$\begin{array}{r|rrrr} -3 & 6 & 13 & -14 & 3 \\ & & -18 & 15 & -3 \\ \hline & 6 & -5 & 1 & 0 \end{array}$$

$$f(x) = (x+3)(6x^2 - 5x + 1) = \boxed{(x+3)(3x-1)(2x-1)}$$

4. (2 points) Find a polynomial function of least degree having only real coefficients with zeros 2 and $1 + i$.

Note: By Conjugate Zeros Thm, $1 - i$ is also a zero.

$$\begin{aligned} f(x) &= (x-2)(x-(1+i))(x-(1-i)) \\ &= (x-2)(x-1-i)(x-1+i) \\ &= (x-2)(x^2 - x + ix - x + 1 - ix + i^2) \\ &= (x-2)(x^2 - 2x + 2) \\ &= x^3 - 2x^2 + 2x - 2x^2 + 4x - 4 = \boxed{x^3 - 4x^2 + 6x - 4} \end{aligned}$$

5. Let $f(x) = x^3 - 2x^2 - 13x - 10$.

a. (1 point) List all possible rational zeros of $f(x)$.

$\pm 1, \pm 2, \pm 5, \pm 10$

b. (2 points) Find all rational zeros.

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -13 & -10 \\ & & 1 & -1 & -14 \\ \hline & 1 & -1 & -14 & -24 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -13 & -10 \\ & & -1 & 3 & 10 \\ \hline & 1 & -3 & -10 & 0 \end{array} \leftarrow \text{So, } -1 \text{ is a zero}$$

$$\begin{aligned} f(x) &= (x+1)(x^2-3x-10) \\ &= (x+1)(x-5)(x+2) \end{aligned}$$

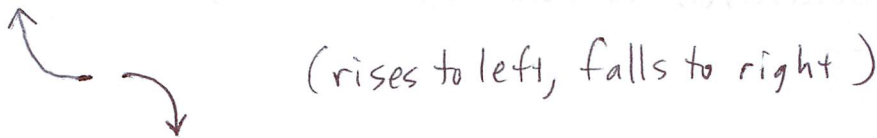
\downarrow \downarrow
 5 -2

Zeros: $-1, 5, -2$

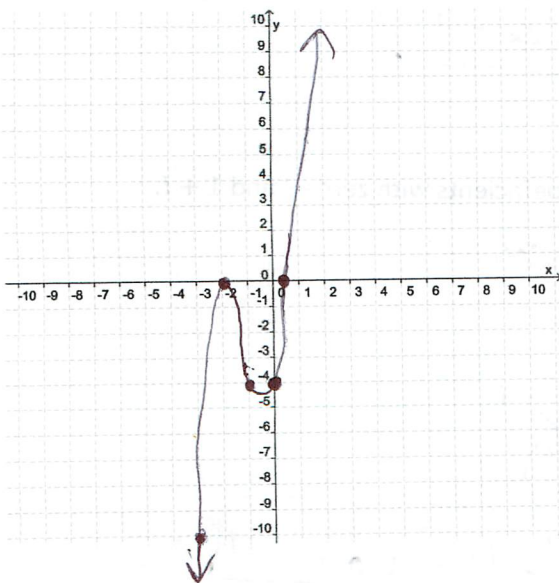
c. (1 point) Factor $f(x)$.

$f(x) = \boxed{(x+1)(x-5)(x+2)}$

6. (1 point) Describe the end behavior of $f(x) = -4x^3 + 3x^2 - 1$. (Note: You can either use the arrows that the book uses, or describe the end behavior in words.)



7. (2 points) Graph $f(x) = (3x - 1)(x + 2)^2$



~~Ques~~ x-intercepts:
 $(3x-1)(x+2)^2 = 0$
 \downarrow \downarrow
 $x = \frac{1}{3}$ $x = -2$

x	f(x)
-3	-10
1	18
-1	-4

y-intercept:
 $f(0) = (3(0) - 1)(0 + 2)^2$
 $= (-1)(2)^2$
 $= -4$