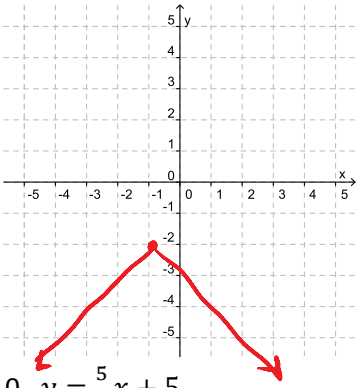
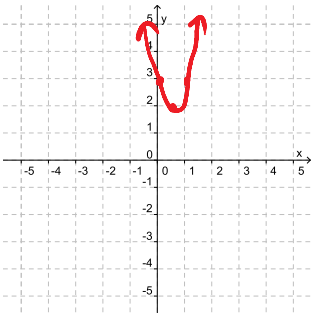
Math 130

Final Exam Review Exercises (Answer Key)

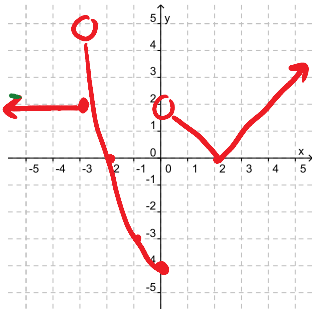
1. $\emptyset$ (or $\{\}$)
2. $\left\{\frac{5}{2}, 4\right\}$
3. $\left\{-\frac{80}{27}, -4\right\}$
4. $\left\{-\frac{3}{2}, 2\right\}$
5. $\left(-\infty, -\frac{1}{2}\right) \cup [6, \infty)$
6. Center: $\left(-\frac{11}{2}, \frac{5}{2}\right)$ Radius: $\frac{\sqrt{146}}{2}$
7. Only the second equation defines y as a function of x.
8. a. $[-7,7]$, b. even, c. y-axis
9. a. (see below), b. $(-\infty, \infty)$, c. $(-\infty, -2]$, d. $(-\infty, -1]$, e. $[-1, \infty)$, f. neither



10. $y = \frac{5}{8}x + 5$
11. $y = x - 8$
12. Symmetric with respect to x-axis
13. a. $\frac{2}{x^2+1}$; $(-\infty, \infty)$, b. $\frac{4}{x^2} + 1$; $(-\infty, 0) \cup (0, \infty)$, c. -4, d. 5, e. -1, f. $-\frac{2}{x(x+h)}$
14. Vertex: $\left(\frac{1}{2}, 2\right)$; Axis: $x = \frac{1}{2}$; Domain: $(-\infty, \infty)$; Range: $[2, \infty)$



15.



16. a. $f(x) = \begin{cases} -\frac{1}{2}x, & \text{if } x < -1 \\ \frac{2}{3}x - 1, & \text{if } x \geq -1 \end{cases}$

b. Domain: $(-\infty, \infty)$; Range: $\left[-\frac{5}{2}, \infty\right)$, c. $x = -1$, d. $[-1, \infty)$, e. $(-\infty, -1)$

17. $3x^2 + 2x + 1 + \frac{8}{x+2}$

18. Yes

19. a. Positive: 2 or 0; Negative: 2 or 0, b. Rising to left and rising to right, c. $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{1}{8}, \pm\frac{3}{8}$

d. $f(x) = (x+1)(x-3)(2x+1)(4x-1)$

20. a. Positive: 3 or 1; Negative: 1, b. Falling to left and falling to right, c. $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

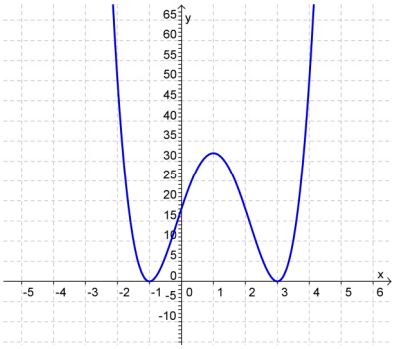
d. $f(x) = -(x-4)(x+3)(x-1-i)(x-1+i)$

21. a. Positive: 2 or 0; Negative: 1, b. Rising to left and falling to right, c. $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}$

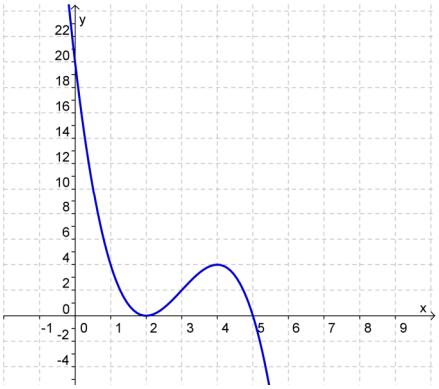
d. $f(x) = -(x-3)(x-1)(2x+1)$

22. $f(x) = x^3 - 3x^2 + x + 5$

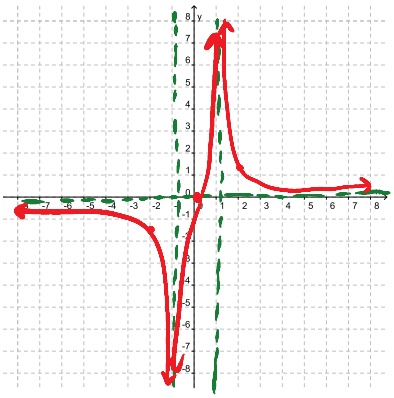
23.



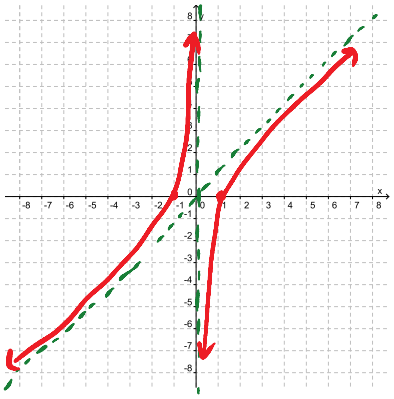
24.



25. a. $x = 1$ and $x = -1$, b. H.A. $y = 0$, c. x-intercept: 0; y-intercept: 0



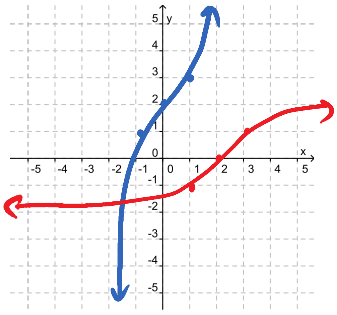
26. a. $x = 0$, b. O.A. $y = x$, c. x-intercepts: 1 and -1; y-intercept: none



27. 1

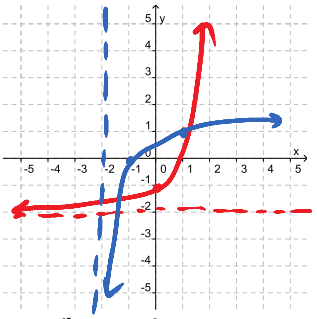
28. a. $f^{-1}(x) = x^3 + 2$

    c. Domain of f: $(-\infty, \infty)$; Range of f: $(-\infty, \infty)$; Domain of f^{-1}: $(-\infty, \infty)$; Range of f^{-1}: $(-\infty, \infty)$



29. a. $f^{-1}(x) = \log_3(x + 2)$

    c. Domain of f: $(-\infty, \infty)$; Range of f: $(-2, \infty)$; Domain of f^{-1}: $(-2, \infty)$; Range of f^{-1}: $(-\infty, \infty)$



30. $\left\{\frac{\ln 2}{\ln 3} - 2\right\}$ (or $\{\log_3 2 - 2\}$)

31. $\left\{\frac{4+\ln 2}{1-\ln 2}\right\}$

32. $\{2\}$

33. $\emptyset$

34. $12\left(\sqrt[120]{\frac{18}{5}} - 1\right) \approx 12.88\%$

35. $\frac{\ln 10}{0.05} \approx 47$ years (rounded up to next year, since at 46 years, the amount would be less than $2000)

36. $2\log_7 x + \frac{1}{4}\log_7 y - 3\log_7 z$

37. $\{(2 - 2y, y)\}$

38. $\{(2,1,-1)\}$

39. $\begin{bmatrix} -2 & -22 \\ 2 & -8 \\ 0 & 12 \end{bmatrix}$

40. a. $2x + 3.5y + 8z$, b. $x \geq 0; y \geq 0; z \geq 0; 4x + 8y + 7 < 10; x + 5y + 9 \geq 8; 15x + 11.7y + 0.4z \geq 10$

41. -31

42. 155
43. $\frac{3}{16}$
44. $\frac{117}{16}$
45. -0.9
46. 85
47. Diverges (since common ratio is 2)
48. Converges to 1 (note that you just showed that $0.\overline{9} = 1$)
49. $81x^4 - 216x^3y^{\frac{1}{2}} + 216x^2y - 96xy^{\frac{3}{2}} + 16y^2$
50. $-57344x^5y^3$
51. $n = 1$: $2 = 2(2-1) = 2 \cdot 1 = 2$ ✓

Suppose $n = k$, so $2 + 2^2 + 2^3 + \cdots + 2^k = 2(2^k - 1)$.

(Now show that $2 + 2^2 + 2^3 + \cdots + 2^k + 2^{k+1} = 2(2^{k+1} - 1)$ is true.)

$$\begin{aligned} LHS &= 2 + 2^2 + 2^3 + \cdots + 2^k + 2^{k+1} \\ &= 2\left(2^k - 1\right) + 2^{k+1} \\ &= 2^{k+1} - 2 + 2^{k+1} \\ &= 2 \cdot 2^{k+1} - 2 \\ &= 2(2^{k+1} - 1) \end{aligned}$$

Thus, by mathematical induction, $2 + 2^2 + 2^3 + \cdots + 2^n = 2(2^n - 1)$ is true for every positive integer n.