Math 130

Final Exam Review Exercises

1. Solve $\sqrt{x+3}-\sqrt{3x+10}=1$.

2. Solve $\frac{13}{x^2+10}=\frac{2}{x}$.

3. Solve $(x+3)^{-\frac{2}{3}}-2(x+3)^{-\frac{1}{3}}=3$

4. Solve $6x^{-2}=2-x^{-1}$.

5. Solve the rational inequality and write the solution set in interval notation.
$\frac{x+7}{2x+1}\leq 1$

6. Find the center and radius of the circle whose graph is given by the equation:
$3x^2+3y^2+33x-15y=0$

7. Which of the following relations define y as a function of x?
$x=\frac{1}{3}y^2$
$y=-\frac{4}{x}$

8. Consider the function: $f(x)=\sqrt{49-x^2}$
   a. What is the domain of $f(x)$?
   b. Is $f(x)$ even, odd, or neither?
   c. Is the graph symmetric with respect to the x-axis, the y-axis, the origin, or none of these?

9. Consider the function: $f(x)=-|x+1|-2$
   a. Graph $f(x)$.
   b. What is the domain of $f(x)$?
   c. What is the range of $f(x)$?
   d. Find the interval(s) where $f(x)$ is increasing.
   e. Find the interval(s) where $f(x)$ is decreasing.
   f. Is $f(x)$ even, odd, or neither?

10. Find the equation of the line passing through $(0,5)$ and perpendicular to $8x+5y=3$. Write the equation in slope-intercept form.

11. Find the equation of the line passing through $(3,-5)$ and parallel to $y=x+4$. Write the equation in slope-intercept form.

12. Without graphing, determine whether the following equation has a graph that is symmetric with respect to the x-axis, the y-axis, the origin, or none of these.
$x=|y|+1$

13. Suppose $f(x) = x^2 + 1$ and $g(x) = \frac{2}{x}$.

a. Find $(g \circ f)(x)$ and its domain.

b. Find $(f \circ g)(x)$ and its domain.

c. Find $(g - f)(-1)$.

d. Find $(fg)(2)$.

e. Find $\left(\frac{f}{g}\right)(-1)$.

f. Find $\frac{g(x+h)-g(x)}{h}$

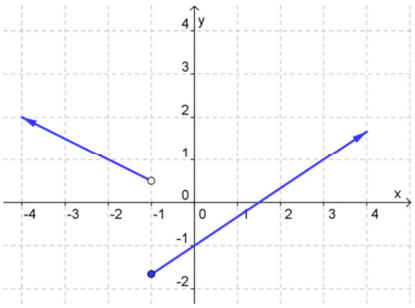
14. Graph the following quadratic equation. Also give the vertex, axis, domain, and range.

$f(x) = 4x^2 - 4x + 3$

15. Graph the following function:

$$f(x) = \begin{cases} 2, & \text{if } x \leq -3 \\ x^2 - 4, & \text{if } -3 < x \leq 0 \\ |x - 2|, & \text{if } x > 0 \end{cases}$$

16. Here's a piecewise-defined function.



a. Give a rule for the function.

b. Give the domain and range of the function.

c. At what x-value(s) is the function discontinuous?

d. Find the interval(s) where $f(x)$ is increasing.

e. Find the interval(s) where $f(x)$ is decreasing.

17. Use synthetic division to perform the following division:

$\frac{3x^3+8x^2+5x+10}{x+2} =$

18. Use synthetic division to decide whether $k = 1 - i$ is a zero of the function $f(x) = x^2 - 2x + 2$.

19. Suppose $f(x) = 8x^4 - 14x^3 - 29x^2 - 4x + 3$.

a. Use Descartes' rule of signs to determine the possible number of positive real zeros and negative real zeros of $f(x)$.

b. Describe the end behavior of $f(x)$.

c. List all possible rational zeros of $f(x)$.

d. Factor $f(x)$ into linear factors.

20. Suppose $f(x) = -x^4 + 3x^3 + 8x^2 - 22x + 24$.
    a. Use Descartes' rule of signs to determine the possible number of positive real zeros and negative real zeros of $f(x)$.
    b. Describe the end behavior of $f(x)$.
    c. List all possible rational zeros of $f(x)$.
    d. Factor $f(x)$ into linear factors, given that $1 + i$ is a zero.

21. Suppose $f(x) = -2x^3 + 7x^2 - 2x - 3$.
    a. Use Descartes' rule of signs to determine the possible number of positive real zeros and negative real zeros of $f(x)$.
    b. Describe the end behavior of $f(x)$.
    c. List all possible rational zeros of $f(x)$.
    d. Factor $f(x)$ into linear factors.

22. Find a polynomial function of least degree having only real coefficients with zeros -1 and $2 - i$.

23. Graph $f(x) = 2(x + 1)^2(x - 3)^2$. Be sure to plot all x-intercepts and the y-intercept.

24. Graph $f(x) = -(x - 2)^2(x - 5)$. Be sure to plot all x-intercepts and the y-intercept.

25. Let $f(x) = \frac{2x}{x^2-1}$
    a. Find any vertical asymptotes.
    b. Find any horizontal or oblique asymptotes.
    c. Find any x-intercepts and the y-intercept.
    d. Graph $f(x)$.

26. Let $f(x) = \frac{x^2-1}{x}$
    a. Find any vertical asymptotes.
    b. Find any horizontal or oblique asymptotes.
    c. Find any x-intercepts and the y-intercept.
    d. Graph $f(x)$.

27. Suppose p varies directly as the square of z, and inversely as r. If $p = \frac{32}{5}$ when $z = 4$ and $r = 10$, find p when $z = 3$ and $r = 36$.

28. Let $f(x) = \sqrt[3]{x - 2}$.
    a. Write an equation for the inverse function in the form $y = f^{-1}(x)$.
    b. Graph f and f^{-1} on the same axes.
    c. Give the domain and range of f and f^{-1}.

29. Let $f(x) = 3^x - 2$.
    a. Write an equation for the inverse function in the form $y = f^{-1}(x)$.
    b. Graph f and f^{-1} on the same axes.
    c. Give the domain and range of f and f^{-1}.

30. Solve the following equation.

$\frac{3^{2x+1}}{3^{x-1}} = 2$

31. Solve the following equation.

$2^{x+1} - e^{x-4} = 0$

32. Solve the following equation.

$\log_2 x + \log_2(x+2) = 3$

33. Solve the following equation.

$\ln x - 4\ln 3 = \ln\left(\frac{1}{5}x\right)$

34. Find the required annual interest rate for $5000 to grow to $18000 if interest is compounded monthly for 10 years.

35. How many years would it take for $200 to grow to $2000 at 5% compounded continuously?

36. Use the properties of logarithms to rewrite the following expression:

$\log_7 \frac{x^2\sqrt[4]{y}}{z^3}$

37. Solve the system. State whether it is inconsistent or has infinitely many solutions. If the system has infinitely many solutions, write the solution set with y arbitrary.

$5x + 10y = 10$

$x + 2y = 2$

38. Use matrices and the Gauss-Jordan method to solve the system.

$2x + 4y + 4z = 4$

$x + 3y + z = 4$

$-x + 3y + 2z = -1$

39. (3 points) Perform the following operations.

$$\left(-2\begin{bmatrix} 1 & 3 \\ -1 & -2 \\ 0 & 2 \end{bmatrix}\right)\begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix}$$

40. Suppose the following foods have the following nutritional content per ounce:

	Bread	**Milk**	**Cheese**
Cost	$2	$3.50	$8
Protein (in grams)	4	8	7
Fat (in grams)	1	5	9
Carbohydrates (in grams)	15	11.7	0.4

You want to know how many ounces of each kind of food would be required make a meal from the three foods with less than 10 grams of protein, at least 10 grams of carbohydrates, and at least 8 grams of fat. You also want your cost to be minimized.

a. Write the objective function.
b. List all constraints.

41. Find a_{20} for the arithmetic sequence with $a_2 = 5$ and $a_5 = -1$.

42. Find S_{10} for the arithmetic sequence $2, 5, 8, 11, \ldots$.

43. Find a_5 for the geometric sequence with first term 3 and common ratio $-\frac{1}{2}$.

44. Find S_4 for the geometric sequence $\frac{3}{4}, -\frac{1}{2}, \frac{1}{3}, \ldots$.

45. Write the terms for the series. Evaluate the sum, if possible.

$\sum_{k=1}^{3} f(x_k)\Delta x$ if $f(x) = (x-2)^3, x_1 = 0, x_2 = 1, x_3 = 2,$ and $\Delta x = 0.1$

46. Use the summation properties to find the sum.

$\sum_{i=1}^{5} (i^2 + 2i)$

47. Evaluate the series if it converges. If the series diverges, say so.

$\frac{1}{12} + \frac{1}{6} + \frac{1}{3} + \frac{2}{3} + \cdots$

48. Evaluate the series if it converges. If the series diverges, say so.

$0.9 + 0.09 + 0.009 + 0.0009 + \cdots$

49. Use the Binomial Theorem to expand the following expression.

$\left(3x - 2\sqrt{y}\right)^4$

50. Find the sixth term of $(4x - y)^8$.

51. Use mathematical induction to prove that $2 + 2^2 + 2^3 + \cdots + 2^n = 2(2^n - 1)$ is true for every positive integer n.