Math 130

7.5 – Mathematical Induction

Consider the following statement:
S_n: $1 + 3 + 5 + \cdots + (2n - 1) = n^2$

Is it true for every positive integer n? Let's look at the first few cases:
S_1: $1 = 1^2$
S_2: $1 + 3 = 2^2$
S_3: $1 + 3 + 5 = 3^2$
S_4: $1 + 3 + 5 + 7 = 4^2$

To prove it's true for every positive integer n, we can use **mathematical induction**.
There are two parts to induction:

1. Show S_1 is true.
2. Show that if S_k is true, then S_{k+1} is also true.

Let's do this for our above example.
1. S_1: $1 = 1^2$, which is true.
2. Suppose S_k is true. Then, $1 + 3 + 5 + \cdots + (2k - 1) = k^2$.
   (Goal: show that S_{k+1}: $1 + 3 + 5 + \cdots + (2k - 1) + (2(k + 1) - 1) = (k + 1)^2$ is true.)

$$
\begin{aligned}
& 1 + 3 + 5 + \cdots + (2k - 1) + (2(k + 1) - 1) \\
&= k^2 + (2(k + 1) - 1) \\
&= k^2 + 2k + 1 \\
&= (k + 1)^2
\end{aligned}
$$

Thus, by the principle of mathematical induction, S_n: $1 + 3 + 5 + \cdots + (2n - 1) = n^2$ is true for every positive integer n.

Ex 1.

Let S_n represent the statement $2 + 5 + 8 + \cdots + (3n - 1) = \frac{n(3n+1)}{2}$. Prove that S_n is true for every positive integer n.

Ex 2.

Prove that if x is a real # between 0 and 1, then for every positive integer n, $0 < x^n < 1$.

Note: Sometimes, we'll need to start the induction at S_3, or S_7, etc.

Ex 3.

Let S_n represent the statement $4^n > 4n$. Show that S_n is true for all values of n such that $n \geq 2$.

Q: A bus driver was heading down a street in Walnut. He went right past a stop sign without stopping, went the wrong way on a one-way street, and then went on the left side of the road past a cop car. The cop did nothing, because he didn't break any traffic laws. Why not?