Math 130

7.2 – Arithmetic Sequences and Series

Arithmetic Sequences

An **arithmetic sequence** is one where you add a fixed # to get the next term.

ex: $1, 7, 13, 19, 25, \ldots$ is an arithmetic sequence. The **common difference** d is ____.

Ex 1.

Find the common difference, d, for the following arithmetic sequence.
$-14, -8, -2, 4, 10, \ldots$

Ex 2.

Find the first five terms of the arithmetic sequence with $a_1 = 1.5$ and $d = -0.5$.

In general, for arithmetic sequences, $a_n = a_1 + (n-1)d$

Ex 3.

Find a_{16} and a_n for the arithmetic sequence $23, 20, 17, 14, \ldots$
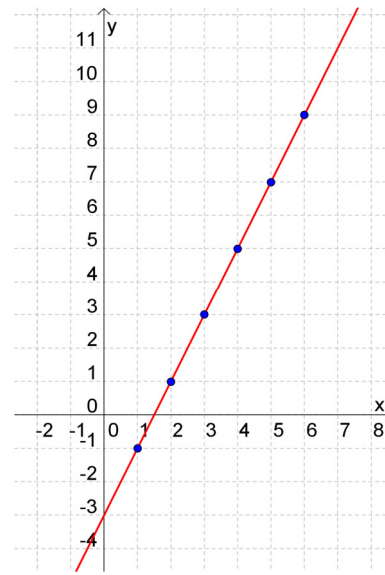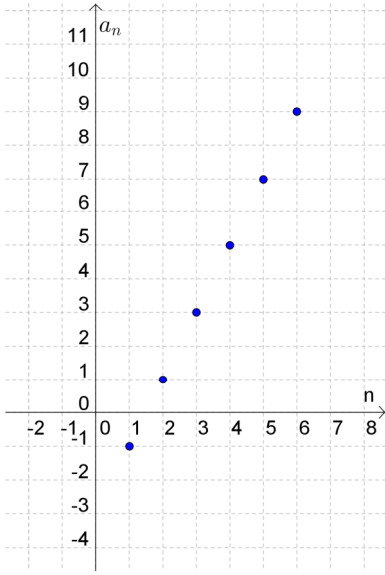
Ex 4.

Find a_{10} and a_n for the arithmetic sequence having $a_4 = 18$ and $a_5 = 22$.
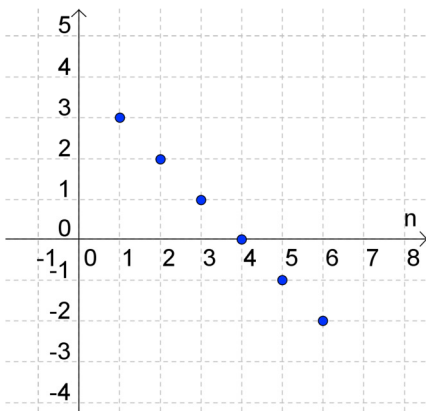
Ex 5.

Suppose an arithmetic sequence has $a_6 = 9$ and $a_{15} = -36$. Find a_1.

Let's connect sequences with their graphs:

Ex 6.

Find a formula for the nth term of the sequence a_n shown below.

What are the domain and range of this sequence?

Arithmetic Series

An **arithmetic series** is the sum of the terms of an arithmetic sequence.

Here's an arithmetic series: $2 + 5 + 8 + 11 + 14 + 17$

And here's an interesting way to figure out the sum:

$$
\begin{array}{c}
2 + 5 + 8 + 11 + 14 + 17 \\
\underline{17 + 14 + 11 + 8 + 5 + 2} \leftarrow \text{Backwards} \\
\underbrace{19 + 19 + 19 + 19 + 19 + 19}_{6 \text{ terms}} = 6 \cdot 19 = 114
\end{array}
$$

So, $2 + 5 + 8 + 11 + 14 + 17 = \frac{114}{2} = 57$

In general, the sum of the first n terms of an arithmetic sequence is:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

(Note: if a_n is not given, you can use our friendly formula: $a_n = a_1 + (n-1)d$)

Ex 7.

Evaluate S_{21} for the arithmetic sequence $48, 44, 40, 36, \dots$

Ex 8.

Evaluate the sum of the first 200 positive integers.

Ex 9.

The sum of the first 15 terms of an arithmetic sequence is 345. If $a_{15} = 65$, find a_1 and d.

Ex 10.

$$\sum_{i=1}^{20} (2i + 5)$$

Ex 11.

$$\sum_{k=5}^{10} (8 - 2k)$$

Q: There is a word in the English language in which the first two letters signify a male, the first three letters signify a female, the first four signify a great man, and the whole word, a great woman. What is the word?