Math 130

7.1 – Sequences and Series

Sequences

A **sequence** is an ordered list.
ex: $1, 3, 5, 7, 9, \ldots$

Mathematically, it is actually a function that takes natural numbers as input.
ex: $f(n) = 2(n-1) + 1$
The list $f(1), f(2), f(3), f(4), f(5), \ldots$ becomes $1, 3, 5, 7, 9, \ldots$

A sequence can be **finite** or **infinite**.
ex: $2, 4, 6, 8, 10$ is finite.
ex: $2, 4, 6, 8, 10, \ldots$ is infinite.

Rather than using $f(n)$, mathematicians usually use a_n for sequences (where n is a natural #).
So, the elements (called **terms**) of a sequence are $a_1, a_2, a_3, a_4, \ldots$

Ex 1.
Write the first five terms of the sequence $a_n = \frac{n+1}{n+2}$

Ex 2.
Write the first three terms of the sequence $a_n = (-1)^{n+1} \cdot n^2$.

Some infinite sequences get closer and closer to a particular real #. These are called **convergent** sequences, and are said to **converge** to that real #.

ex: $a_n = \frac{1}{n}$

If a_n is not convergent, it is **divergent**.

ex: $a_n = n^2$

Sequences that use earlier terms to define later terms are called **recursive sequences**.

ex: $a_1 = 2, \;\; a_n = 3 \cdot a_{n-1} + n - 1 \;\; (\text{if } n > 1)$

Ex 3.

Write the first four terms of the recursive sequence:

$a_1 = 2$

$a_n = 3 \cdot a_{n-1} + n - 1 \;\; (\text{if } n > 1)$

Series and Summation Notation

If you add up the terms of a sequence, you get a **series**.

ex: $1, 3, 5, 7, 9$ is a sequence

$1 + 3 + 5 + 7 + 9$ is a series

Finite series have the form:

$$S_n = a_1 + a_2 + a_3 + \cdots + a_n = \sum_{i=1}^{n} a_i$$

Infinite series have the form:

$$S_n = a_1 + a_2 + a_3 + \cdots + a_n + \cdots = \sum_{i=1}^{\infty} a_i$$

Ex 4.

Evaluate the series:

$$\sum_{k=1}^{4}(k^2 + k + 1)$$

Ex 5.

Write the terms for each series. Evaluate each sum, if possible.

$$\sum_{i=1}^{5} b_i$$

$$\sum_{j=1}^{4}(2x_j - 5) \quad \text{if } x_1 = 3, x_2 = 5, x_3 = 7, x_4 = 9$$

$$\sum_{k=1}^{3} f(x_k)\Delta x \quad \text{if } f(x) = \frac{1}{x}, x_1 = 1, x_2 = 3, x_3 = 5, \text{and } \Delta x = 4$$

Summation Properties

Note: below, c is a constant.

$\sum_{i=1}^{n} c = nc$	$\sum_{i=1}^{n} i = 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$
$\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$	$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$
$\sum_{i=1}^{n} (a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$	$\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$

Ex 6.

Use the summation properties to find each sum.

$$\sum_{i=1}^{50} 12$$

Q: What is harder to catch the faster you run?

$$\sum_{i=1}^{20} 2i$$

$$\sum_{i=1}^{10} (3i^2 + 5)$$

$$\sum_{i=1}^{5} (i^3 + 2i - 8)$$