Math 130

5.7 – Properties of Matrices

Here's a general $m \times n$ matrix:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Square matrix: $\begin{bmatrix} 3 & 0 \\ 1 & -2 \end{bmatrix}$

Row matrix: $\begin{bmatrix} 11 & -24 & 7 \end{bmatrix}$

Column matrix: $\begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix}$

Zero matrix: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ etc. (any matrix with all zero elements)

Matrices are equal if they are same size, and have same elements in same places.

Ex 1.

Find the values of the variables for which each statement is true, if possible.

$$\begin{bmatrix} a & b \\ -5 & 0 \end{bmatrix} = \begin{bmatrix} -3 & 9 \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

Adding/Subtracting Matrices

Ex 2.

Find each sum, if possible.

$$\begin{bmatrix} 3 & -8 \\ -4 & 6 \end{bmatrix} + \begin{bmatrix} 7 & -5 \\ 10 & -6 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix} + \begin{bmatrix} -7 \\ 4 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \end{bmatrix} + \begin{bmatrix} -1 \\ 5 \end{bmatrix}$$

Ex 3.

Find each difference, if possible.

$$\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 3 \\ -6 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 6 & -3 \end{bmatrix} - \begin{bmatrix} 5 & 11 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 \\ -9 & 24 \end{bmatrix} - \begin{bmatrix} -1 & -3 \\ 5 & -5 \\ 1 & 0 \end{bmatrix}$$

Multiplying Matrices

To contrast with a matrix, a real # is called a **scalar**. To multiply a scalar and a matrix, multiply each element of the matrix by the scalar.

Ex 4.

$$-3\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$$

Now let's look at multiplying a matrix by a matrix.

Ex 5.

$$\begin{bmatrix} 2 & 1 & -3 \\ -1 & 0 & 4 \end{bmatrix}\begin{bmatrix} 5 & 0 \\ 2 & 6 \\ -2 & 3 \end{bmatrix}$$

Notice that # of columns of first matrix must equal # of rows of second matrix.

Also, if we multiply an $m \times n$ matrix by an $n \times p$ matrix, we'll get an $m \times p$ matrix.

Ex 6.

Suppose A is a 2×5 matrix and B is a 5×3 matrix.

Can AB be calculated?

If so, what size is it?

Can BA be calculated?

Ex 7.

Suppose $A = \begin{bmatrix} 3 & -4 & 5 \\ 1 & 0 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 \\ -3 & 8 \end{bmatrix}$.

Find AB, if possible.

Find BA, if possible.

Ex 8.

Suppose $A = \begin{bmatrix} 2 & -5 \\ -6 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -4 \\ 2 & 3 \end{bmatrix}$.

Find AB.

Find BA.

So, matrix multiplication is not commutative (that is, in general $AB \neq BA$).

Properties of Matrices

If A and B are matrices of the same size, and c and d are scalars, then…

$(c + d)A = cA + dA$

$c(A + B) = cA + cB$

$c(A)d = cd(A)$

$(cd)A = c(dA)$

Also, if A, B, and C are such that the products and sums below exist, then…

$(AB)C = A(BC)$

$A(B + C) = AB + AC$

$(B + C)A = BA + CA$

Q: What question can someone ask all day long, always get completely different answers, and yet all the answers could be correct?