Math 130

5.2 – Matrix Solution of Linear Systems

The important part of a system of linear equations is the coefficients. Matrices focus on this.

A **matrix** (plural **matrices**) is a rectangular array of numbers enclosed in brackets. Each number is called an **element** of the matrix. Here is a matrix with 3 rows and 2 columns:

$$\begin{bmatrix} 3 & -2 \\ 1 & 0 \\ -7 & 9 \end{bmatrix}$$

The following system of linear equations is turned into a matrix called an **augmented matrix**.

$$\begin{matrix} x - 3y - 2z = -3 \\ 3x + 2y - z = 12 \\ -x - y + 4z = 3 \end{matrix} \rightarrow \left[\begin{array}{ccc|c} 1 & -3 & -2 & -3 \\ 3 & 2 & -1 & 12 \\ -1 & -1 & 4 & 3 \end{array}\right]$$

Because augmented matrices represent systems of linear equations, you can…
…switch any two rows.
…multiply any row by a nonzero real #.
…combine two rows by addition and replace one of the combined rows with the result.

If we can reduce an augmented matrix to the following form, then we've solved the system.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{array}\right]$$

This is the goal of the **Gauss-Jordan method**, and is called **reduced-row echelon form**.

Gauss-Jordan Method

Ex 1.

Use the Gauss-Jordan method to solve the system.

$4x - 3y = 14$

$x - 2y = 1$

Ex 2.

Use the Gauss-Jordan method to solve the system.

$x + y - 4z = 10$

$2x - 3y + z = 7$

$3x - y - z = 12$

Ex 3.

Use the Gauss-Jordan method to solve the system:

$2x - 3y = 7$

$-6x + 9y = 0$

Ex 4.

Use the Gauss-Jordan method to solve the system. Write the solution set with z arbitrary.

$2x + y + z = 5$

$3x + 2y - z = -8$

Q: What is the word that everybody always says wrong?