Math 130

5.1 – Systems of Linear Equations

Systems of Linear Equations

Recall: A **linear equation** has the form $a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$

Suppose we have two linear equations:

$4x - 3y = 14$

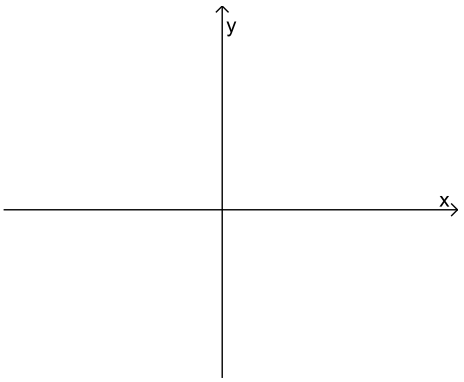
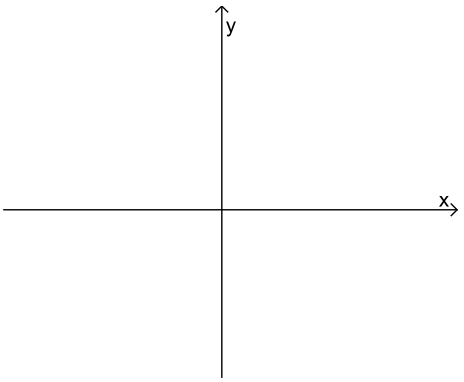
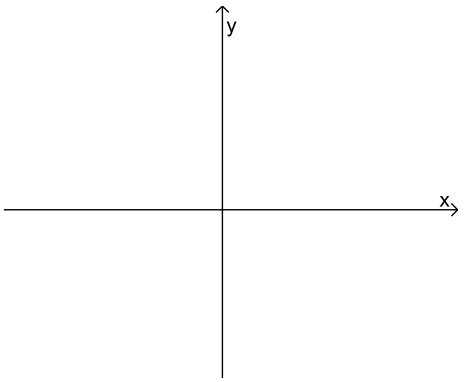
$x - 2y = 1$

Together, they make a **system of linear equations** (also called a **linear system**).

Solutions to a system of linear equations must satisfy *all* equations.

For example, $(5,2)$ is a solution to the above system since it satisfies both equations.

We can visualize systems of linear equations in two unknowns (x and y) by graphing.

		
Exactly one solution	No solution	Infinitely many solutions

Substitution Method

To solve a system of equations, we can solve one equation for x or y, and then substitute the expression we get for x or y into the other equation. This makes an equation with one variable that we can solve. After that, we can find the other variable.

Ex 1.

Solve the system.

$4x - 3y = 14$

$x - 2y = 1$

Elimination Method

Another way to solve linear systems. Also called the addition method.

Ex 2.

$2x + 3y = -1$

$3x - 2y = 18$

If get a *false* statement (like $0 = 9$), then system is **inconsistent** (i.e. no solution).
If get a *true* statement (like $0 = 0$), then system is **dependent** (i.e. infinitely many solutions).

Ex 3.
Solve the system:
$5x - 2y = 4$
$-10x + 4y = 7$

Ex 4.
Solve the system:
$x = 4y - 8$
$5x - 20y = -40$

Solving Linear Systems with Three Unknowns (Variables)

Here's an example of a **linear equation in three variables**: $2x - 3y + z = 4$

Solutions are ordered triples, (x, y, z), which correspond with 3-D points.

To solve, take 2 different pairs of equations and eliminate the same variable from both pairs. Then solve the smaller system.

Ex 5.

Solve the system:

$2x + y - 3z = -5$

$x - 2y + 2z = 3$

$3x + 5y + z = 16$

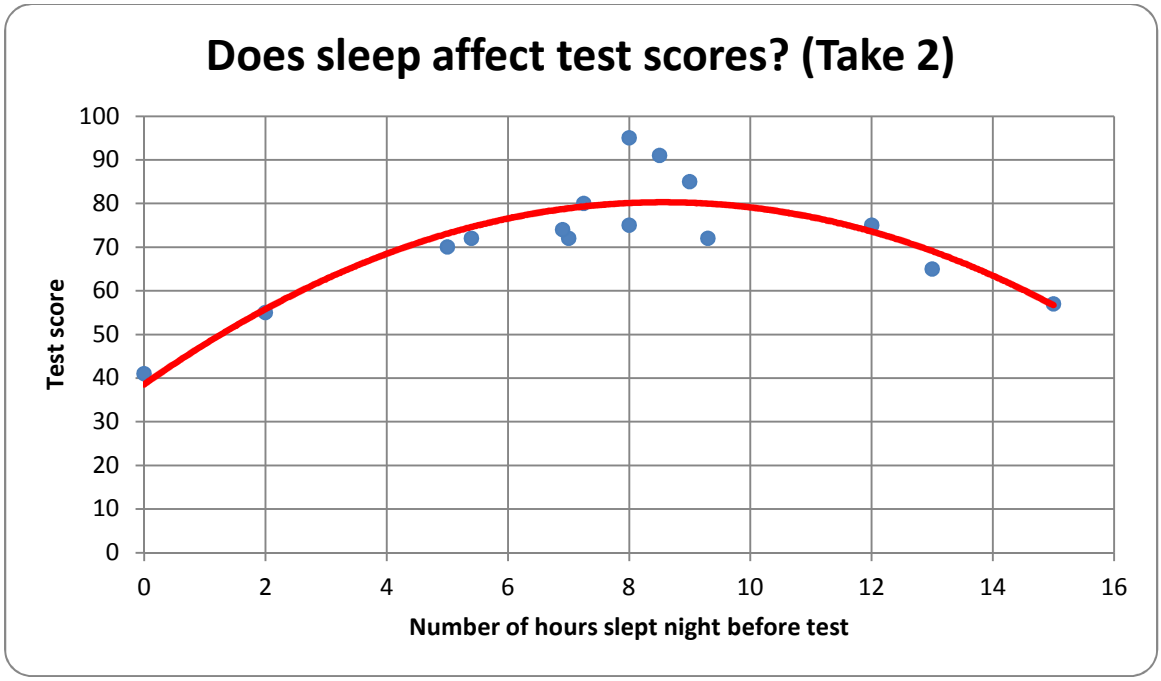
Ex 6.

Solve the system:

$3x + y - 2z = -7$

$5x + 2y + z = -6$

Quadratic Curve Fitting



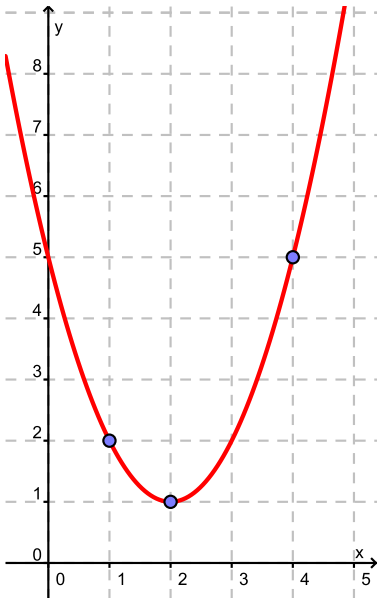
In general, **quadratic functions** have the form: $y = ax^2 + bx + c$ (where $a \neq 0$)

For example: $y = 2x^2 - 3x + 1$

Let's focus on fitting the graph of a quadratic function through just 3 points…

Ex 7.

Suppose your data consists of 3 ordered pairs: $(1,2)$, $(2,1)$, and $(4,5)$. Find a quadratic function to model this data.



Ex 8.

Suppose the following foods have the following nutritional content:

	Calories	**Protein (in grams)**	**Vitamin C (in mg)**
Food A	40	5	30
Food B	200	2	10
Food C	400	4	300

If you want to make a meal from the three foods with exactly 660 calories, 25 grams of protein, and 425 mg of vitamin C, how many ounces of each kind of food should be used?

Q: What has four wheels and flies?