

Math 130

4.4 – Evaluating Logarithms and the Change-of-Base Theorem

Notation:

$\log x = \log_{10} x$ (common logarithm)

$\ln x = \log_e x$ (natural logarithm)

Applications

To measure the acidity or alkalinity of a solution (i.e. the pH), this formula is used:

$$\text{pH} = -\log[\text{H}_3\text{O}^+]$$

To measure the loudness of a sound in decibels, this formula is used:

$$d = 10 \log \frac{I}{I_0}$$

To measure the age of rocks and fossils, logarithms are used.

Looking at stock prices, you're often given an option to view the y-axis on a logarithmic scale.

In computer science, the speed of algorithms often involves logarithms. For example, suppose you have 16 numbers in numerical order in a list. Let's consider two algorithms to search for a given number:

Change-of-Base Theorem

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Ex 1.

Use the Change-of-Base Theorem to rewrite $\log_5 17$ in terms of natural logarithms.

Ex 2.

Use the Change-of-Base Theorem to rewrite $\log_2 0.3$ in terms of common logarithms.

Ex 3.

Use the Change-of-Base Theorem to rewrite $\log_{\sqrt{2}} e$ in terms of natural logarithms.

Q: A man is driving his son to school. They get into an accident and the man dies. The son is rushed to the hospital and when he arrives for emergency surgery the doctor says "I can't operate on this boy, HE'S MY SON!" The boy was not adopted. How is this possible?