Math 130

4.3 – Logarithmic Functions

One-to-one functions have inverses. Let's define the inverse of the exponential function.
To find inverse of $y = a^x$, we switch x and y, and then solve for y:
$x = a^y$

We don't have tools to solve for y, so we just define what's called the **logarithmic function**:
$\boldsymbol{y = \log_a x}$
(Note: Here, a and x are positive, and $a \neq 1$.)

So, $\log_a x$ is the exponent to which you have to raise a in order to get x.

Ex 1.
$\log_2 8 =$

$\log_e \frac{1}{e^3} =$

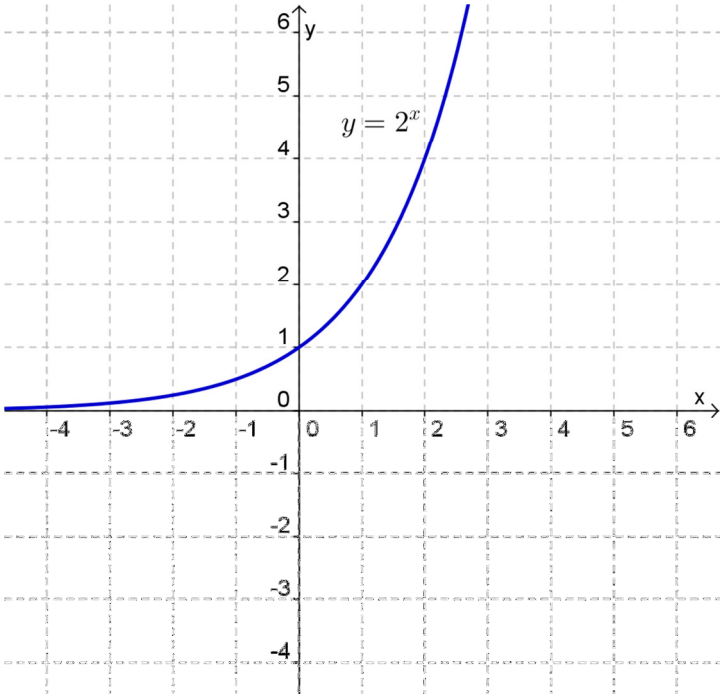
Ex 2.
$\log_x \frac{16}{9} = -2$

$\log_{16} x = \frac{3}{4}$

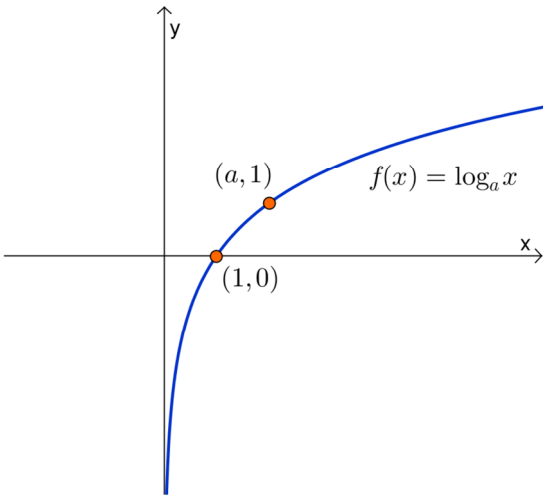
$\log_{36} \sqrt{6} = x$

Graph of Logarithmic Function

Let's graph $y = \log_2 x$, knowing that it's the inverse of 2^x:

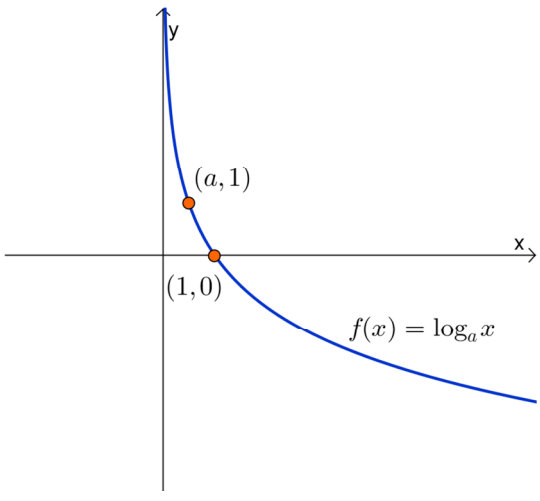


In general, the **logarithmic function with base a** is $\boldsymbol{f(x) = \log_a x}$ (where $a > 0$, $a \neq 1$, $x > 0$).



$\boldsymbol{a > 1}$

ex: $f(x) = \log_3 x$



$\boldsymbol{0 < a < 1}$

ex: $f(x) = \log_{1/3} x$

What is the vertical asymptote?

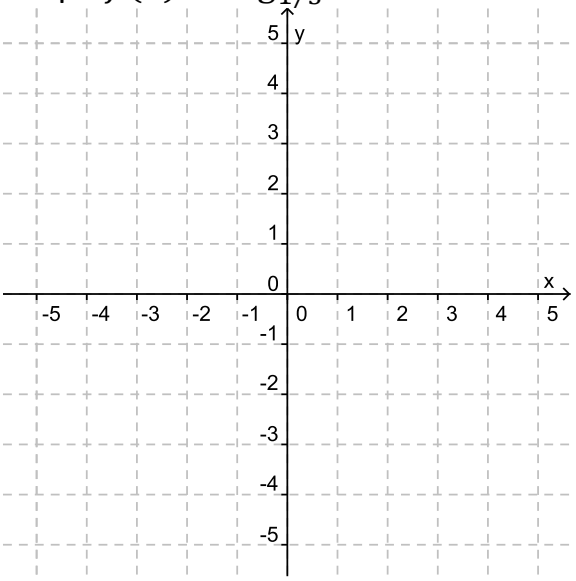
Any horizontal asymptotes?

What is the domain?

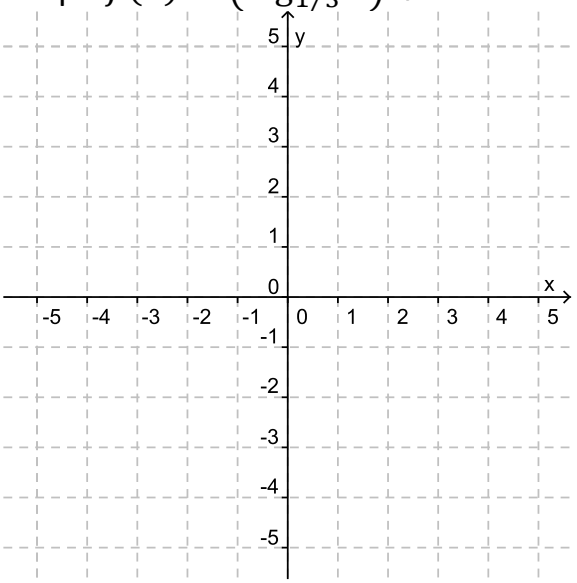
What is the range?

Ex 3.

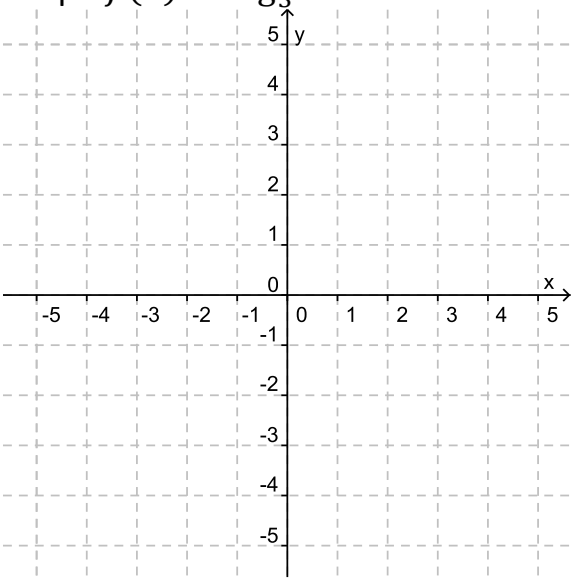
Graph $f(x) = \log_{1/3} x$



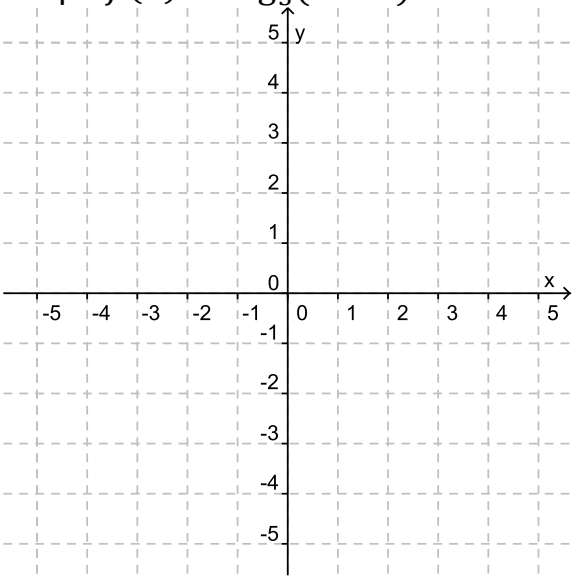
Graph $f(x) = \left(\log_{1/3} x\right) + 1$



Graph $f(x) = \log_3 x$



Graph $f(x) = \log_3(x + 1)$



Properties of Logarithms

Product Property: $\log_a xy = \log_a x + \log_a y$

Quotient Property: $\log_a \frac{x}{y} = \log_a x - \log_a y$

Power Property: $\log_a x^r = r \log_a x$

Also, two more helpful properties are: $\log_a 1 = 0$ and $\log_a a = 1$

Ex 4.

Use properties of logarithms to rewrite (expand) each expression.

$\log_7(8 \cdot 6) =$

$\log_6 \frac{12}{5} =$

$\log_2 \sqrt[3]{9} =$

$\log_b \frac{rs^2t}{u^3v^3} =$

$\log_a \sqrt[5]{r^3} =$

$\log_a \sqrt[m]{\frac{r^3s^2}{t^4}} =$

Ex 5.

Write each expression as a single logarithm with coefficient 1.

$\log_4 x - \log_4 y + \log_4 z =$

$4\log_b r - 5\log_b s =$

$\frac{1}{3}\log_a x + \frac{2}{3}\log_a y - \log_a xy =$

Note: $\boldsymbol{a^{\log_a x} = x}$ and $\boldsymbol{\log_a a^x = x}$

Ex 6.

$7^{\log_7 10} =$

$\log_r r^{k+1} =$

Q: What is it the more you take away the larger it becomes?